DICACIM programme A.Y. 2023–24
Numerical Methods for Partial Differential Equations

Finite Elements discretization for 2d problems

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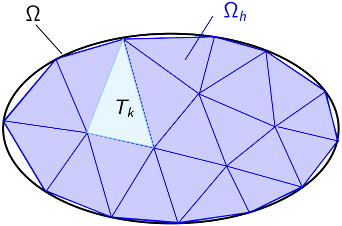
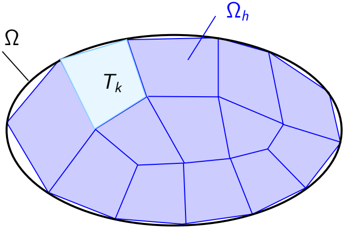
Triangulation of $\Omega \subset \mathbb{R}^2$

Conforming partitions of Ω:



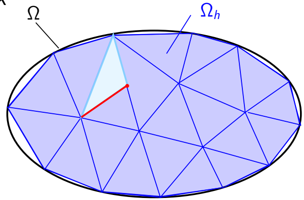


Triangles

Quadrilaterals

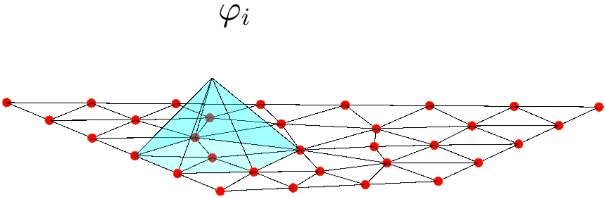
Triangulation: $\mathcal{T}_h = \{T_k, k = 1, \ldots, Ne, T_k \cap T_\ell = \emptyset \textit{ if } k \neq \ell\}$

The discretized domain: $\overline{\Omega}_h = \cup_k \overline{T}_k$

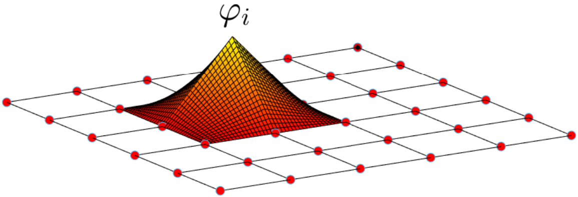


A non–conforming partition of Ω

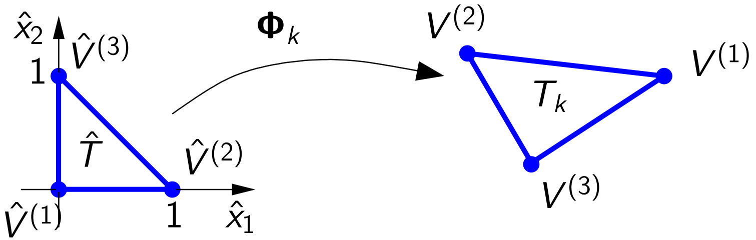
$\mathbb{P}_1$ basis functions

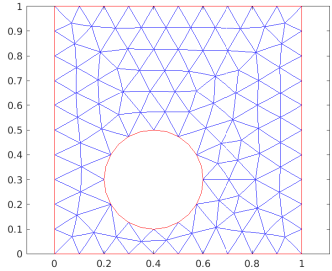


$\mathbb{Q}_1$ basis functions

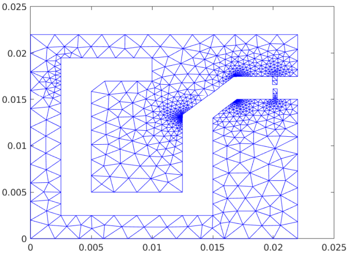


Affine map from $\hat{T}$ to T_k





A regular, quasi uniform mesh



A regular, but not quasi–uniform mesh

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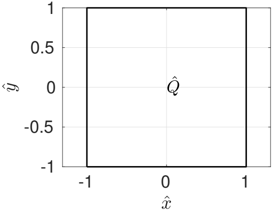
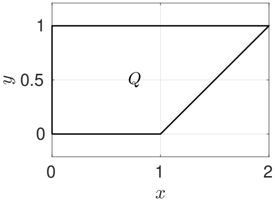
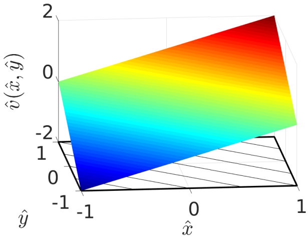
Non–affine map of $\hat{Q}$ to Q_k

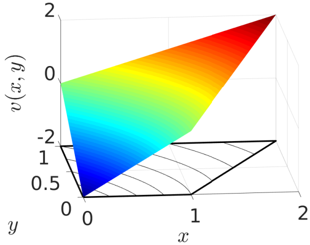




$$\mathbf{\Phi}\left(\begin{bmatrix}\hat{x}\\ \hat{y}\end{bmatrix}\right) = \begin{bmatrix}\frac{\hat{x}+1}{2}\,\frac{\hat{y}+3}{2}\\ \frac{\hat{y}+1}{2}\end{bmatrix}$$

$$\mathbf{\Phi}^{-1}\left(\begin{bmatrix}x\\ y\end{bmatrix}\right) = \begin{bmatrix}\frac{2x-y-1}{y+1}\\ 2y-1\end{bmatrix}$$





$$\hat{v}(\hat{x},\hat{y}) = \hat{x}+\hat{y} \in \mathbb{Q}_1(\hat{Q}) \qquad v(x,y) = \frac{2x-y-1}{y+1} + 2y - 1 \notin \mathbb{Q}_1(Q)$$

$\mathbb{Q}_1 = \{v(x_1,x_2) = a_0 + a_1x_1 + a_2x_2 + a_3x_1x_2,\ a_i \in \mathbb{R}\}$ polynomials of degree 1 w.r.t each variable.

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