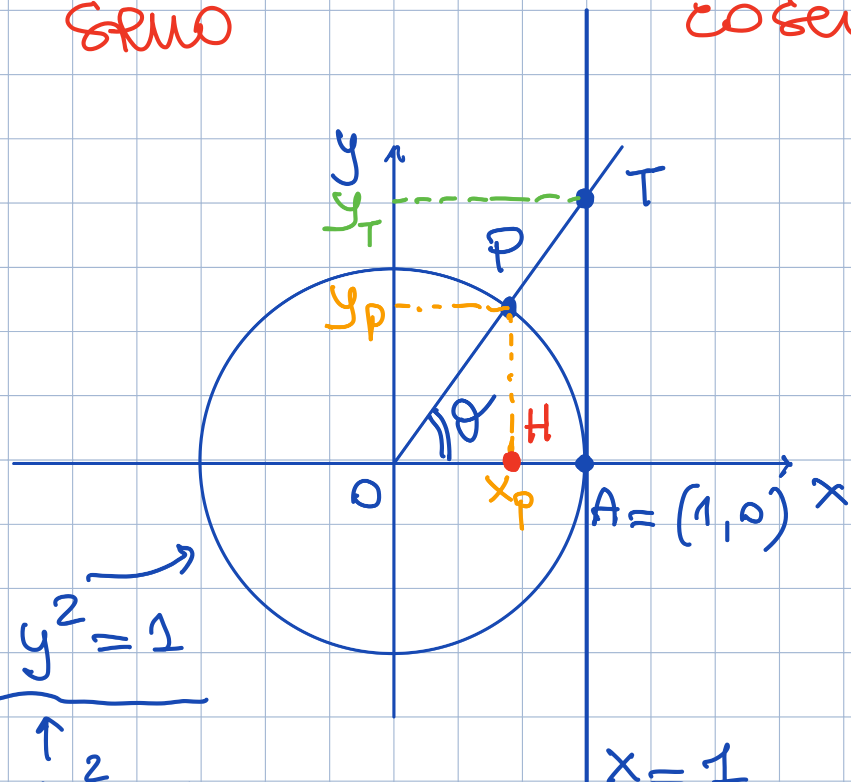
FUNZIONI GONIOMETRICHE

$y = \sin(x)$, $y = \cos(x)$ $y = tg(x)$

seno coseno tangente



$H = (x_P, 0)$ $P = (x_P, y_P)$

$T = (1, y_T)$

Def: $\sin(\theta) = y_P$

$\cos(\theta) = x_P$

$tg(\theta) = y_T$

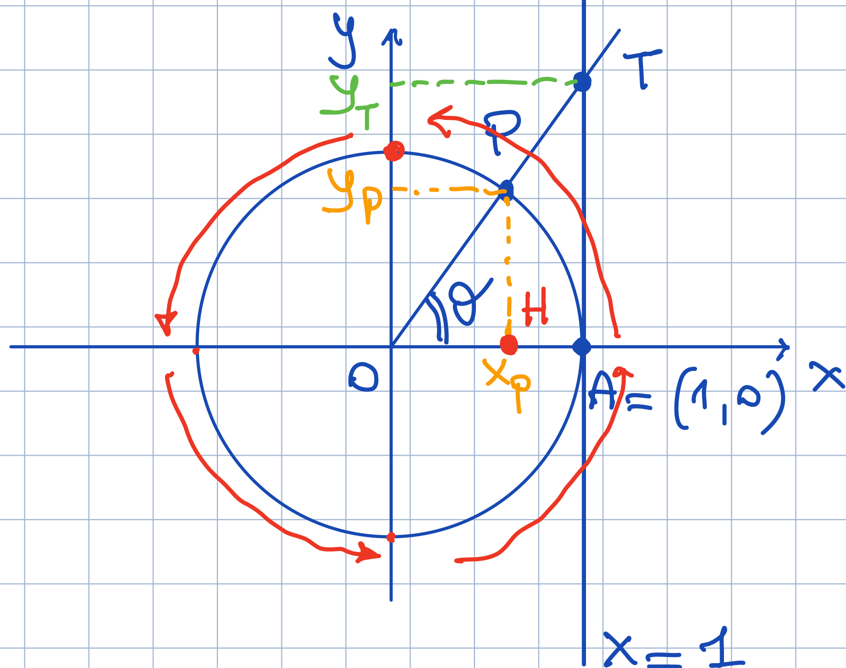
$x^2 + y^2 = 1$

$x_P^2 + y_P^2 = 1$

Relazione fondamentale della goniometria

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$P\hat{O}H$ e $T\hat{O}A$ sono triangoli simili



$$\overline{TA} : \overline{OA} = \overline{PH} : \overline{OH}$$

$$y_T : 1 = y_P : x_P$$

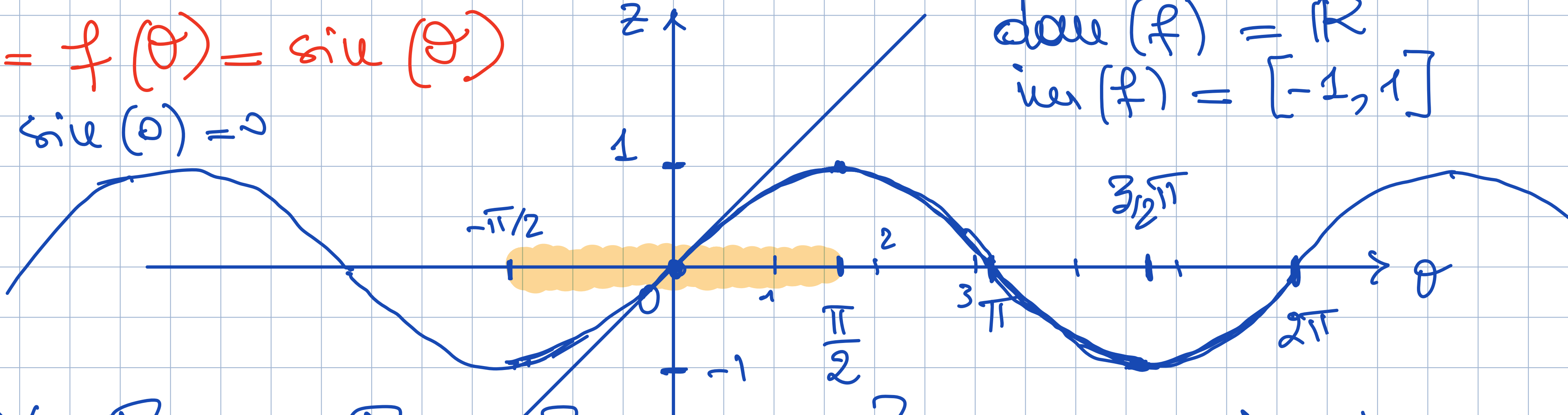
$$tg(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$z = f(\theta) = \sin(\theta)$$

$\sin(\theta) = 0$

dom $(f) = \mathbb{R}$

im $(f) = [-1, 1]$



$X = \mathbb{R}$, $Y = \mathbb{R}$

? f è suri ? no perchè im $(f) \subset Y$

Rendo f suriettiva: $f : \mathbb{R} \to [-1, 1]$ ($Y = \text{im}(f)$)

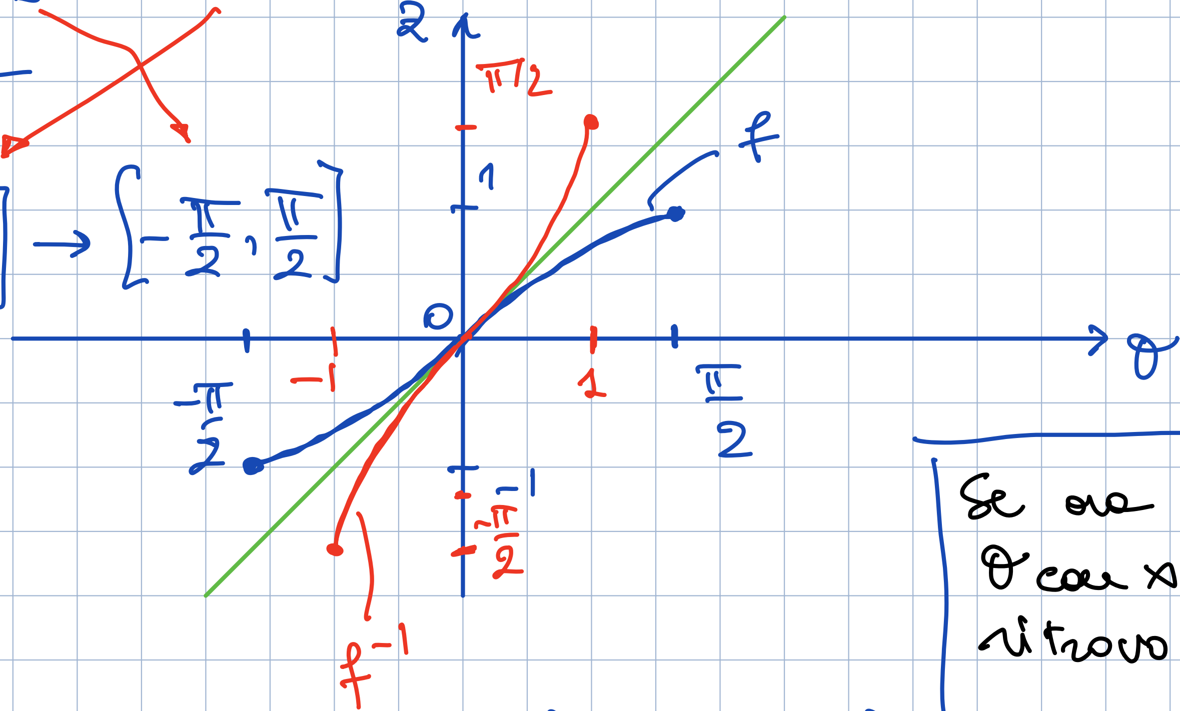
f non è iniettiva, per renderla iniettiva restringo il dominio a $[-\frac{\pi}{2}, \frac{\pi}{2}]$

$f : [-\frac{\pi}{2}, \frac{\pi}{2}] \to [-1, 1]$

è biunivoca

$\Downarrow$

$\exists\, f^{-1} : [-1, 1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$



$z = f^{-1}(\theta) = \text{arcsin}(\theta)$ (arcoseno)

z è l'angolo il cui sin è θ

Se ora rinomino θ con x e z con y ritrovo:

$y = f(x) = \sin(x)$

$y = f^{-1}(x) = \text{arcsin}(x)$

ES:

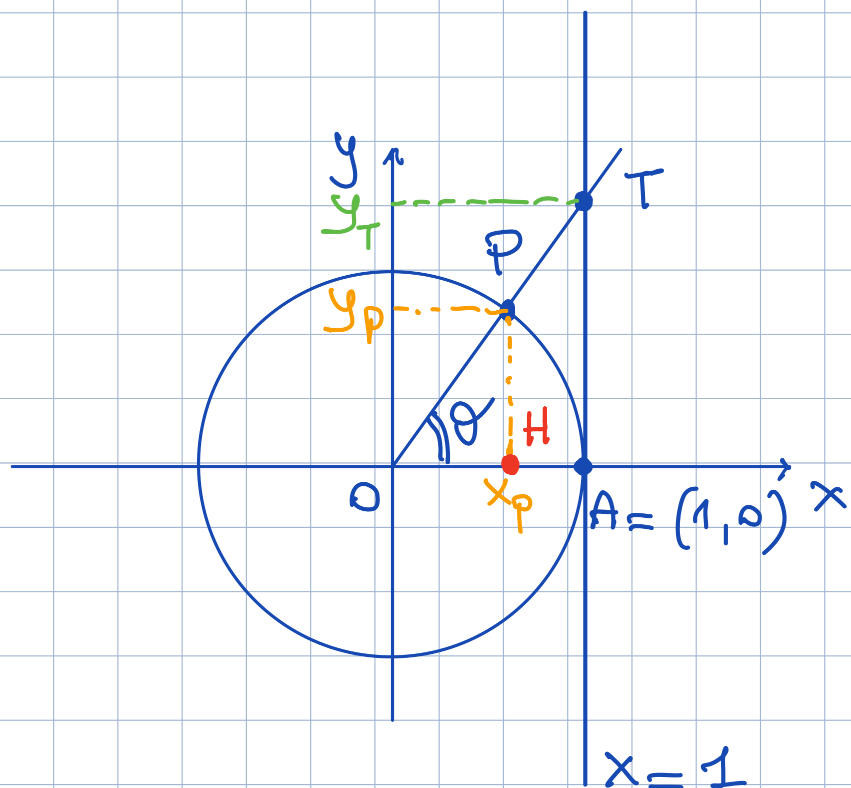
$f(x) = \arcsin(x-3)$

? dom $(f) = \{ x \in \mathbb{R} : -1 \leq x - 3 \leq 1 \}$

$+3 \quad +3 \quad +3$

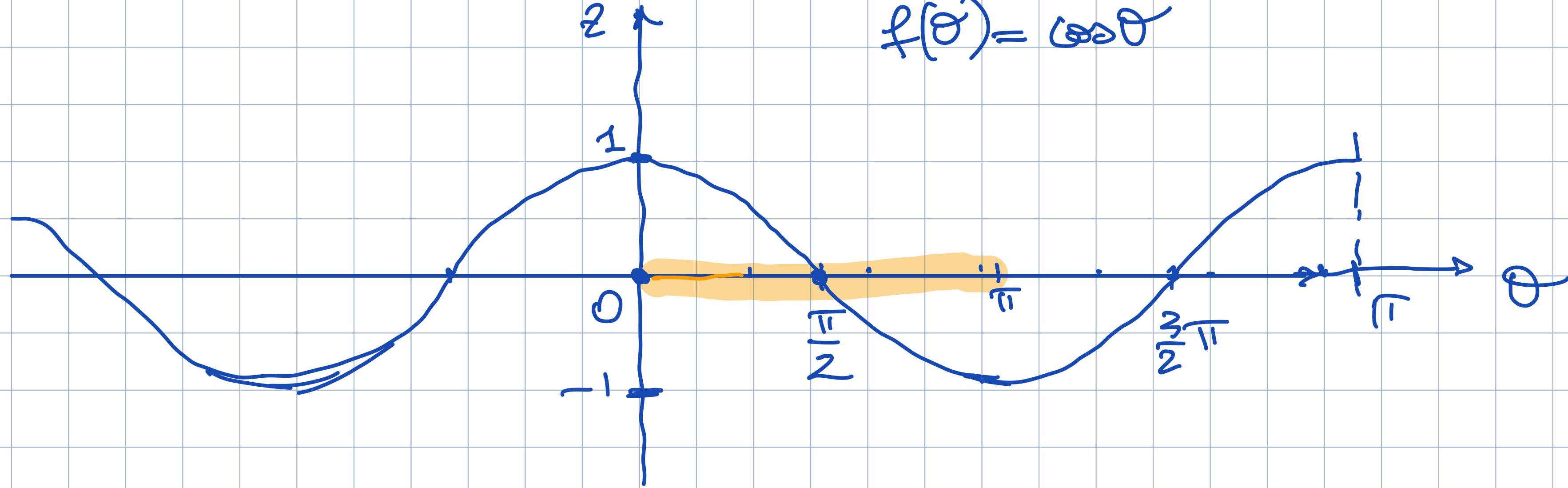
$2 \leq x \leq 4$

$f(\theta) = \cos(\theta)$



$x_P = \cos\theta$

$f(\theta) = \cos\theta$



$\text{dom}(f) = \mathbb{R}$, $\quad \text{im}(f) = [-1, 1]$

per invertire f scelgo dominio $= [0, \pi]$

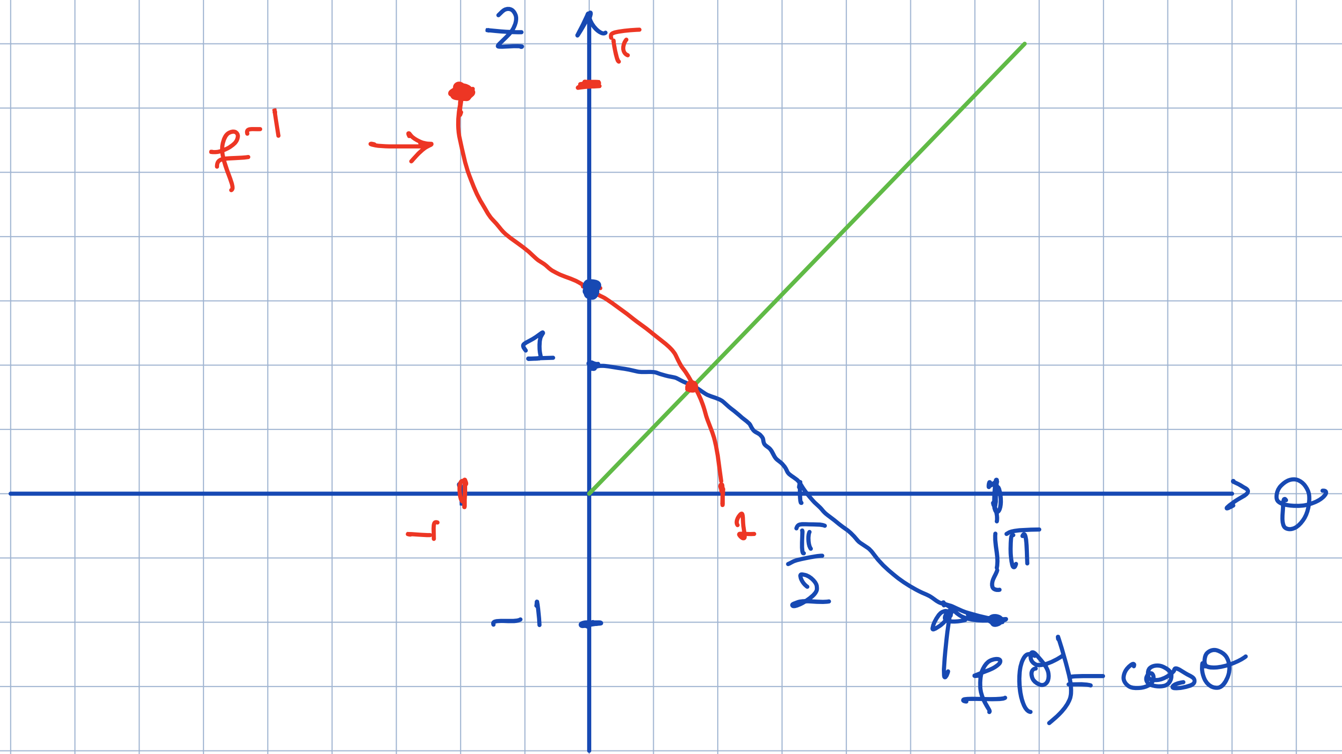
codominio $= \text{im}(f) = [-1, 1]$

$f: [0, \pi] \to [-1, 1] \qquad z = f(\theta) = \cos\theta$

inversa:

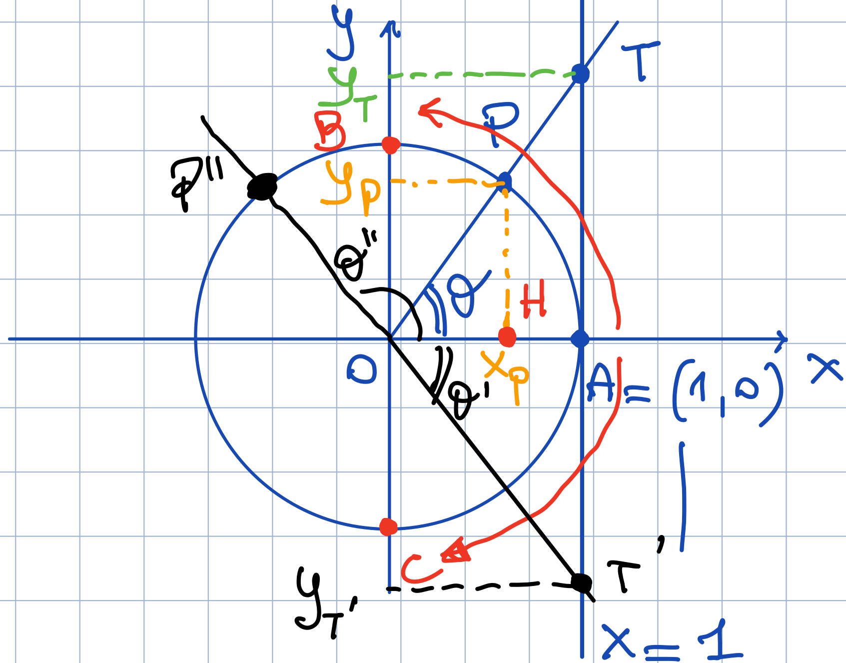
$f^{-1}: [-1, 1] \to [0, \pi] \qquad z = f^{-1}(\theta) = \arccos(\theta)$ arcocoseno

z è l'angolo il cui coseno vale θ



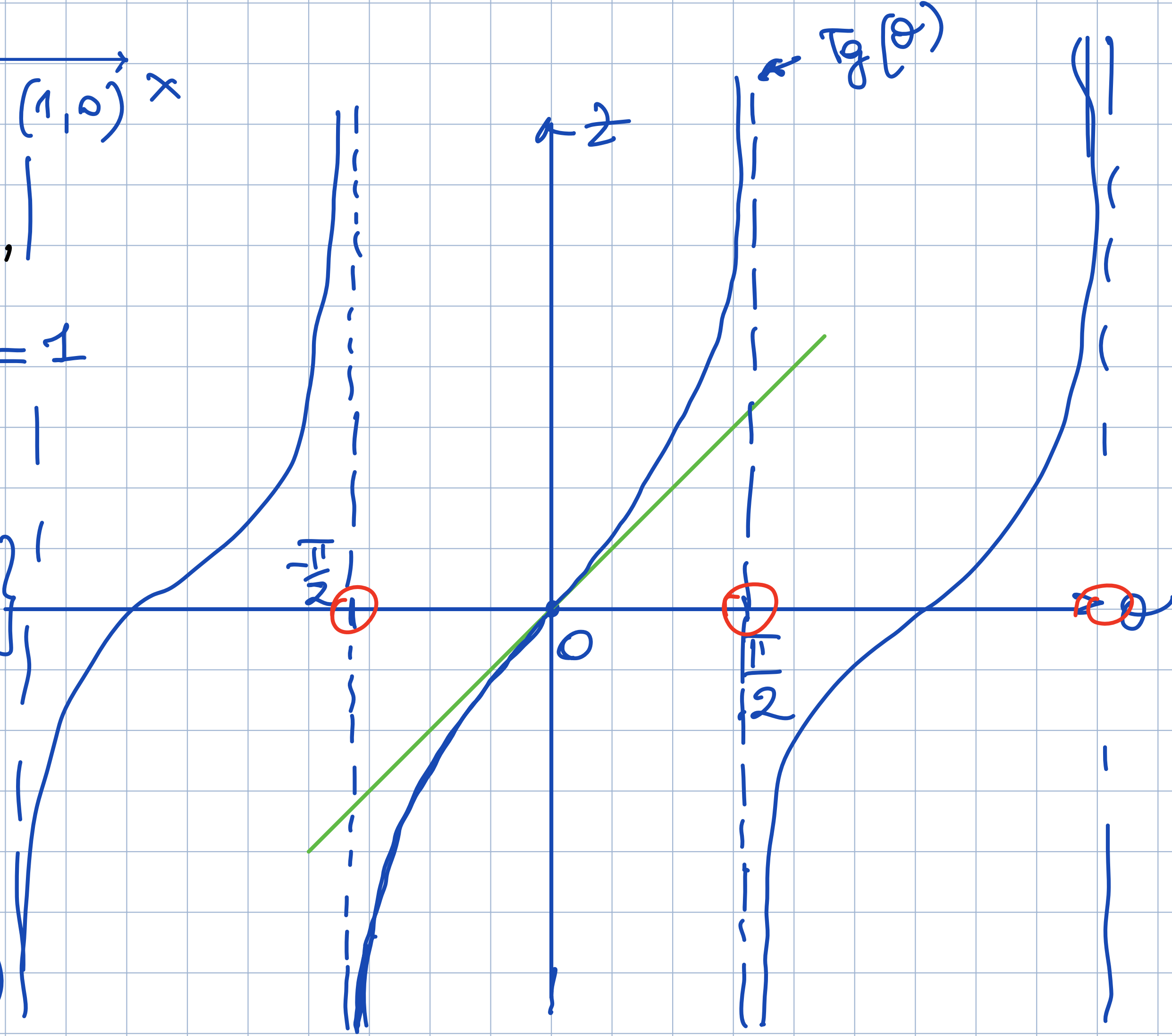
? $\arccos(0) = \frac{\pi}{2}$ (senza periodicità)

$f(\theta) = tg(\theta)$



$f(\theta) = tg(\theta)$

$dom(f) = \{\theta \in \mathbb{R} : \theta = \frac{\pi}{2} + k\pi\}$
$k \in \mathbb{Z}$



$$tg(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

(devo chiedere $\cos(\theta) \neq 0$)

$im(f) = \mathbb{R}$, la sur è automatica ma non ho l'iniettività.

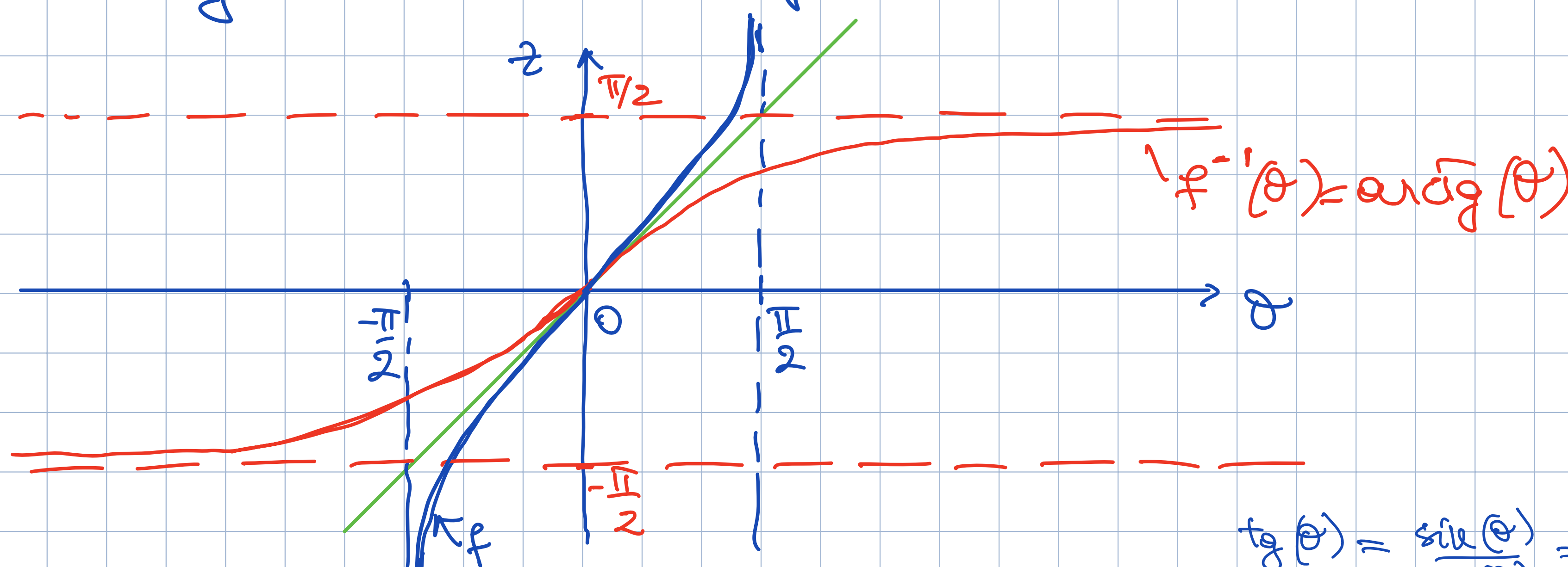
Per poter invertire f, restringo il dom

$$f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \rightarrow \mathbb{R}$$

la funzione inversa $f^{-1} : \mathbb{R} \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

si chiama $z = arctg(\theta)$ arcotangente

z è l'angolo la cui tangente vale θ



? $z = arctg(1) = \frac{\pi}{4}$

$tg(\theta) = \frac{sin(\theta)}{cos(\theta)} = 1$

se $\theta = \frac{\pi}{4}$