Analisi Matematica 1 – Esercitazione del 20 novembre 2025

1. Calcolare il limite $\lim_{x\to 0} \dfrac{\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2}{1-\cos x + x^4}$.

2. Calcolare il limite $\lim_{x\to 0} \left(\dfrac{\sin x}{x}\right)^{\left(\frac{1}{\arctan x}\right)^2}$.

3. (Prova d'esame 3 luglio 2017) Calcolare il limite $\lim_{x\to 0} \dfrac{\log\left(\frac{1+x}{1+3x}\right)+2\sin x}{\cosh(3x)-\cos(3x)}$.

4. (Prova d'esame 3 febbraio 2020) Calcolare il limite $\lim_{x\to 0} \dfrac{\log(\cos(7x^2))}{\sqrt{1+7x^4}-\sqrt[3]{1+7x^4}}$.

5. (Prova d'esame 16 gennaio 2018) Calcolare il limite $\lim_{n\to+\infty} \dfrac{[e^{2n}-\left(\frac{1}{2}\right)^n][(n+1)!-n!]}{(n!-7^n)[\sin\frac{1}{n}-\frac{1}{4}\sin\frac{4}{n}]\sqrt{n^8e^{4n}+\sin\left(\frac{n}{7}\right)}}$.

6. (Prova d'esame 2 luglio 2019) Determinare per quali valori di $\alpha\in\mathbb{R}$ esiste finito il limite
$$\lim_{x\to 0^+} \frac{e^{x\cos x - x}-1}{(2x-\sin(2x))^{7\alpha}}.$$

7. (Prova d'esame 31 agosto 2015) Calcolare il limite
$$\lim_{n\to+\infty} \frac{\cosh\left(\frac{n!}{n^{7n}}\right)-\cos\left(\frac{n!}{n^{7n}}\right)}{1+\log\left(1+\frac{(n+1)!}{(n+1)^{7n+1}}\right)-\exp\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right)}.$$

8. (Test 16 gennaio 2018) Calcolare il limite
$$\lim_{x\to 0} \frac{\sin x - \arctan x}{(e^{x/4}-1+\sinh(x^{10}))\left(\frac{1}{x}\log(1+x^3)+\cos x - 1\right)}.$$

9. Calcolare il limite $\lim_{x\to 0} \dfrac{\cos(2x)+\sin^2 x - \frac{1}{1+x^2}}{e^{x^4}-1}$.

10. (Prova d'esame 25 giugno 2014) Sia $\alpha\in\mathbb{R}^-$. Calcolare il limite
$$\lim_{x\to+\infty} \frac{x\sin\left(\frac{1}{\sqrt{x}}\right)-\sqrt{x}}{\arctan\left(\frac{x^\alpha}{6}\right)}.$$

Calcolare il limite $\displaystyle\lim_{x\to 0} \frac{\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2}{1-\cos x+x^4}$

$\to 0$

$\to 0$

$\to 1$

$\to 0$

Se $\alpha \in \mathbb{R}$,

$$(1+t)^\alpha = 1+\alpha t + \frac{\alpha(\alpha-1)}{2}t^2 + o(t^2) \quad \text{per } t\to 0$$

Nel caso $\alpha = \frac{1}{3}$

$$(1+t)^{1/3} = 1+\frac{1}{3}t+\frac{1}{2}\cdot\frac{1}{3}\cdot\left(\frac{1}{3}-1\right)t^2+o(t^2) =$$

$$= 1+\frac{1}{3}t-\frac{1}{9}t^2+o(t^2) \quad \text{per } t\to 0$$

$$\sqrt[3]{1+\sin x} = 1+\frac{1}{3}\sin x-\frac{1}{9}\sin^2 x+o(\sin^2 x) \quad \text{per } x\to 0$$

NUMERATORE

$$\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2 = \cancel{1}+\cancel{\frac{1}{3}\sin x}-\frac{1}{9}\sin^2 x+o(\sin^2 x)$$

$$+\cancel{1}-\cancel{\frac{1}{3}\sin x}-\frac{1}{9}\sin^2 x+o(\sin^2 x)-\cancel{2} =$$

$$= -\frac{2}{9}\sin^2 x+o(\sin^2 x)$$

$$\sim -\frac{2}{9}\sin^2 x$$

$$\boxed{\begin{gathered}\sin x \sim x\\ \text{per } x\to 0\end{gathered}}$$

$$\sim -\frac{2}{9}x^2 \quad \text{per } x\to 0$$

DENOMINATORE

$$\cos x = 1-\frac{1}{2}x^2+o(x^3)$$

$$1-\cos x+x^4=\cancel{1}-\cancel{1}+\frac{1}{2}x^2+o(x^3)+\underbrace{x^4}_{=\,o(x^3)}=$$

$$=\frac{1}{2}x^2+o(x^3)\quad \text{per } x\to 0$$

$$\left(1-\cos x+x^4\right)\sim\frac{1}{2}x^2\quad \text{per } x\to 0$$

$$\lim_{x\to 0}\frac{\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2}{1-\cos x+x^4}=\lim_{x\to 0}\frac{-\frac{2}{9}\cancel{x^2}}{\frac{1}{2}\cancel{x^2}}=-\frac{4}{9}$$

Variante 1 $$\lim_{x\to 0}\frac{\sqrt[3]{1+\sin x}+\sqrt[3]{1-\sin x}-2}{1-\cos x+4x}=\lim_{x\to 0}\frac{-\frac{2}{9}x^{\cancel{2}}}{4\cancel{x}}=0$$

$$1-\cos x+4x=\underbrace{\frac{1}{2}x^2}_{=\,o(x)}+\underbrace{o(x^3)}_{=\,o(x)}+4x=4x+o(x)\quad \text{per } x\to 0$$

Variante 2 $$\lim_{x\to 0}\frac{\sqrt[3]{1+\sin x}-\sqrt[3]{1-\sin x}-2}{1-\cos x+x^2}=\lim_{x\to 0}\frac{-\frac{2}{9}\cancel{x^2}}{\frac{3}{2}\cancel{x^2}}=-\frac{2}{9}\cdot\frac{2}{3}=-\frac{4}{27}$$

$$1-\cos x+x^2=\frac{1}{2}x^2+o(x^3)+x^2=\frac{3}{2}x^2+o(x^3)\sim\frac{3}{2}x^2\quad \text{per } x\to 0$$

Calcolare il limite $\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\left(\frac{1}{\arctan x}\right)^2}$

$\frac{\sin x}{x} \to 1$ $\quad \arctan x \to 0$ $\quad \left(\frac{1}{\arctan x}\right)^2 \to +\infty$

$\boxed{\text{NOTAZIONE} \quad \exp(x) = e^x \quad \forall x \in \mathbb{R}}$

$$\left(\frac{\sin x}{x}\right)^{\frac{1}{\arctan^2 x}} = \exp\left(\frac{1}{\arctan^2 x}\cdot \log\left(\frac{\sin x}{x}\right)\right)$$

$$\sin x = x - \frac{x^3}{6} + o(x^4) \quad \text{per } x\to 0$$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{6} + o(x^3) \quad \text{per } x\to 0$$

$$\log(1+t) = t - \frac{t^2}{2} + o(t^2) \quad \text{per } t\to 0$$

$$\log\left(\frac{\sin x}{x}\right) = \log\left(1 - \frac{x^2}{6} + o(x^3)\right) =$$

$$= -\frac{x^2}{6} + o(x^3) + o\left(-\frac{x^2}{6} + o(x^3)\right) =$$

$$= -\frac{x^2}{6} + o(x^3) + o(x^2) =$$

$$= -\frac{x^2}{6} + o(x^2) \quad \text{per } x\to 0$$

Quindi $\log\left(\frac{\sin x}{x}\right) \sim -\frac{x^2}{6}$ per $x\to 0$

$\arctan x \sim x$ per $x \to 0$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{\arctan^2 x}} = \lim_{x\to 0} \exp\left(\frac{1}{\arctan^2 x}\cdot\log\left(\frac{\sin x}{x}\right)\right) =$$

$$= \lim_{x\to 0} \exp\left(\frac{1}{\cancel{x^2}}\cdot\left(-\frac{\cancel{x^2}}{6}\right)\right) =$$

$$= e^{-\frac{1}{6}} = \frac{1}{\sqrt[6]{e}}$$

(03/07/2017)

Calcolare $\displaystyle\lim_{x \to 0} \frac{\log\left(\frac{1+x}{1+3x}\right) + 2\sin x}{\cosh(3x) - \cos(3x)}$

$$\cosh t = 1 + \frac{t^2}{2} + o(t^3) \quad \text{per } t \to 0$$

$$\cos t = 1 - \frac{t^2}{2} + o(t^3) \quad \text{per } t \to 0$$

DENOMINATORE

$$\cosh(3x) - \cos(3x) = \cancel{1} + \frac{(3x)^2}{2} + o(x^3) - \cancel{1} + \frac{(3x)^2}{2} + o(x^3)$$

$$= 9x^2 + o(x^3) \quad \text{per } x \to 0$$

Quindi $\cosh(3x) - \cos(3x) \sim 9x^2$ per $x \to 0$

NUMERATORE

$$\log(1+t) = t - \frac{t^2}{2} + o(t^2) \quad \text{per } t \to 0$$

$$\sin t = t - \frac{t^3}{6} + o(t^4) \quad \text{per } t \to 0$$

$$\log\left(\frac{1+x}{1+3x}\right) + 2\sin x = \log(1+x) - \log(1+3x) + 2\sin x$$

$$= \cancel{x} - \frac{x^2}{2} + o(x^2) - \cancel{3x} + \frac{(3x)^2}{2} + o(x^2) +$$

$$+ \cancel{2x} \overbrace{- \frac{2x^3}{6}}^{=o(x^2)} + \underbrace{o(x^4)}_{=o(x^2)} =$$

$$= 4x^2 + o(x^2) \quad \text{per } x \to 0$$

Osservazione. È sufficiente lo sviluppo $\sin x = x + o(x^2)$ per $x \to 0$

Quindi $\log\left(\frac{1+x}{1+3x}\right)+2\sin x \sim 4x^2$ per $x\to 0$

$$\lim_{x\to 0}\frac{\log\left(\frac{1+x}{1+3x}\right)+2\sin x}{\cosh(3x)-\cos(3x)} = \lim_{x\to 0}\frac{4\cancel{x^2}}{9\cancel{x^2}} = \frac{4}{9}$$

(03/02/2020)

Calcolare $\lim_{x\to 0}\frac{\log(\cos(7x^2))}{\sqrt{1+7x^4}-\sqrt[3]{1+7x^4}} = (\Delta)$

$\cos t = 1-\frac{t^2}{2}+o(t^3)$ per $t\to 0$

$\log(1+t)=t+o(t)$ per $t\to 0$

NUMERATORE

$$\cos(7x^2) = 1-\frac{(7x^2)^2}{2}+o(x^6) =$$

$$= 1-\frac{49}{2}x^4+o(x^6) \quad \text{per } x\to 0$$

$$\log(\cos(7x^2)) = \log\left(1-\frac{49}{2}x^4+o(x^6)\right) =$$

$$= -\frac{49}{2}x^4+o(x^6) + o\left(-\frac{49}{2}x^4+o(x^6)\right) =$$

$$= -\frac{49}{2}x^4+o(x^6)+o(x^4) =$$

$$= -\frac{49}{2}x^4+o(x^4)$$

$$\sim -\frac{49}{2}x^4 \quad \text{per } x\to 0$$

DENOMINATORE

$$(1+t)^{1/2} = 1 + \frac{1}{2}t + o(t) \quad \text{per } t \to 0$$

$$(1+t)^{1/3} = 1 + \frac{1}{3}t + o(t) \quad \text{per } t \to 0$$

$$\sqrt{1+7x^4} - \sqrt[3]{1+7x^4} = \cancel{1} + \frac{7}{2}x^4 + o(x^4) - \cancel{1} - \frac{7}{3}x^4 + o(x^4)$$

$$= \frac{7}{6}x^4 + o(x^4)$$

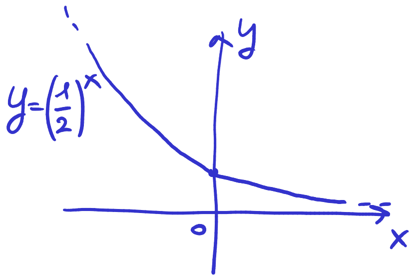
$$\sim \frac{7}{6}x^4 \quad \text{per } x \to 0$$

$$(\Delta) = \lim_{x\to 0} \frac{-\frac{49}{2}\cancel{x^4}}{\frac{7}{6}\cancel{x^4}} = -\frac{\overset{7}{\cancel{49}}}{\underset{1}{\cancel{2}}} \cdot \frac{\overset{3}{\cancel{6}}}{\underset{1}{\cancel{7}}} = -21$$

(16/01/2018)

Calcolare $\displaystyle\lim_{n\to+\infty} \frac{\left[e^{2n}-\left(\frac{1}{2}\right)^n\right]\cdot\left[(n+1)!-n!\right]}{(n!-7^n)\left(\sin\frac{1}{n}-\frac{1}{4}\sin\frac{4}{n}\right)\sqrt{n^8e^{4n}+\sin\left(\frac{n}{7}\right)}} = \circledast$

- $e^{2n}-\underbrace{\left(\frac{1}{2}\right)^n}_{\to 0} \sim e^{2n}$ per $n\to+\infty$



- $(n+1)!-n! = (n+1)!\left[1-\frac{n!}{(n+1)!}\right] =$

$$= (n+1)!\left(1-\frac{1}{n+1}\right) \sim (n+1)! \quad \text{per } n\to+\infty$$

- $n!-7^n \sim n!$ per $n\to+\infty$ (gerarchia degli infiniti)

- $n^8e^{4n}+\underbrace{\sin\left(\frac{n}{7}\right)}_{\text{non ha limite per } n\to+\infty} = n^8e^{4n}\Bigg(1+\underbrace{\frac{\overbrace{\sin\left(\frac{n}{7}\right)}^{\text{succ. limitata}}}{\underbrace{n^8e^{4n}}_{\to+\infty}}}_{\to 0}\Bigg) \sim n^8e^{4n}$ per $n\to+\infty$

$$\sqrt{n^8e^{4n}+\sin\left(\frac{n}{7}\right)} \sim \sqrt{n^8e^{4n}} = n^4e^{2n} \quad \text{per } n\to+\infty$$

- $\sin t = t-\frac{t^3}{6}+o(t^4)$ per $t\to 0$

$$\sin\frac{1}{n}-\frac{1}{4}\sin\frac{4}{n} = \cancel{\frac{1}{n}}-\frac{1}{6}\cdot\left(\frac{1}{n}\right)^3+o\left(\frac{1}{n^3}\right)+$$

$$-\frac{1}{4}\left(\frac{\cancel{4}}{n}-\frac{1}{6}\cdot\left(\frac{4}{n}\right)^3+o\left(\frac{1}{n^3}\right)\right)=$$

$$=-\frac{1}{6n^3}+o\left(\frac{1}{n^3}\right)+\frac{16}{6n^3}+o\left(\frac{1}{n^3}\right)=$$

$$=\frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{6}}n^3}+o\left(\frac{1}{n^3}\right)\sim\frac{5}{2n^3}\quad \text{per } n\to+\infty$$

$$(*)=\lim_{n\to+\infty}\frac{\cancel{e^{2n}}\cdot(n+1)!}{n!\cdot\frac{5}{2\cancel{n^3}}\cdot n^{\cancel{4}}\cancel{e^{2n}}}=$$

$$=\lim_{n\to+\infty}\frac{(n+1)\cancel{n!}}{\cancel{n!}\cdot\frac{5}{2}n}=$$

$$=\lim_{n\to+\infty}\frac{2}{5}\cdot\frac{n+1}{n}=\frac{2}{5}$$

$\sim n$

$\downarrow$ 1

(02/07/2019)

Determinare per quali $\alpha \in \mathbb{R}$ esiste finito

$$\lim_{x\to 0^+} \frac{e^{x\cos x - x} - 1}{(2x-\sin(2x))^\alpha} = (\square)$$

NUMERATORE

$$x\cos x - x = x\left(1 - \frac{1}{2}x^2 + o(x^3)\right) - x =$$

$$= x\left(\cancel{1} - \frac{1}{2}x^2 + o(x^3) - \cancel{1}\right) =$$

$$= -\frac{1}{2}x^3 + o(x^4) \quad \text{per } x\to 0$$

$$e^t = 1 + t + o(t) \quad \text{per } t\to 0$$

$$e^{x\cos x - x} - 1 = e^{-\frac{1}{2}x^3 + o(x^4)} - 1 =$$

$$= \cancel{1} - \frac{1}{2}x^3 + \underbrace{o(x^4)}_{=o(x^3)} + \underbrace{o\left(-\frac{1}{2}x^3 + o(x^4)\right)}_{=o(x^3)} - \cancel{1} =$$

$$= -\frac{1}{2}x^3 + o(x^3) \sim -\frac{1}{2}x^3 \quad \text{per } x\to 0$$

DENOMINATORE

$$2x - \sin(2x) = 2x - 2x + \frac{(2x)^3}{6} + o(x^4) =$$

$$= \frac{4}{3}x^3 + o(x^4) \sim \frac{4}{3}x^3 \quad \text{per } x\to 0$$

$$(2x-\sin(2x))^\alpha \sim \frac{4^\alpha}{3^\alpha}x^{3\alpha} \quad \text{per } x\to 0$$

$$(\square) = \lim_{x\to 0^+} \frac{-\frac{1}{2}x^3}{\frac{4^\alpha}{3^\alpha}x^{3\alpha}} =$$

$$= \lim_{x\to 0^+} \left(-\frac{1}{2}\right)\frac{3^\alpha}{4^\alpha}\cdot x^{3-3\alpha}$$

$$\lim_{x\to 0^+} x^\beta = \begin{cases} 0 & \text{se } \beta>0 \\ 1 & \text{se } \beta=0 \\ +\infty & \text{se } \beta<0 \end{cases}$$

Il limite esiste finito se e solo se

$$3-3\alpha \geq 0$$

$$3\alpha \leq 3$$

$$\alpha \leq 1$$

$$(\square) = \lim_{x\to 0^+} \left(-\frac{1}{2}\right)\frac{3^\alpha}{4^\alpha}x^{3-3\alpha} = \begin{cases} 0 & \text{se } \alpha<1 \\ -\frac{3}{8} & \text{se } \alpha=1 \\ -\infty & \text{se } \alpha>1 \end{cases}$$

(31/08/2015)

gerarchia degli infiniti

Calcolare $\lim_{n\to+\infty} \dfrac{\cosh\left(\frac{n!}{n^{7n}}\right) - \cos\left(\frac{n!}{n^{7n}}\right)}{1+\log\left(1+\frac{(n+1)!}{(n+1)^{7n+1}}\right) - \exp\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right)} = (1)$

$\frac{n!}{n^{7n}} \to 0$, $\frac{(n+1)!}{(n+1)^{7n+1}} \to 0$

NUMERATORE

$$\cosh t - \cos t = \cancel{1} + \frac{t^2}{2} + o(t^3) - \cancel{1} + \frac{t^2}{2} + o(t^3)$$
$$= t^2 + o(t^3) \quad \text{per } t \to 0$$

$$t = \frac{n!}{n^{7n}}$$

$$\cosh\left(\frac{n!}{n^{7n}}\right) - \cos\left(\frac{n!}{n^{7n}}\right) \sim \frac{(n!)^2}{n^{14n}} \quad \text{per } n \to +\infty$$

DENOMINATORE

$$1 + \log(1+t) - e^t = \cancel{1} + \cancel{t} - \frac{t^2}{2} + o(t^2) - \left(\cancel{1} + \cancel{t} + \frac{t^2}{2} + o(t^2)\right) =$$
$$= -t^2 + o(t^2) \quad \text{per } t \to 0$$

$$t = \frac{(n+1)!}{(n+1)^{7n+1}}$$

$$1 + \log\left(1 + \frac{(n+1)!}{(n+1)^{7n+1}}\right) - \exp\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right) =$$
$$= -\frac{[(n+1)!]^2}{(n+1)^{14n+2}} + o\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right)^2 \sim -\frac{[(n+1)!]^2}{(n+1)^{14n+2}} \quad \text{per } n \to +\infty$$

$$(\Delta) = \lim_{n\to+\infty} \frac{\dfrac{(n!)^2}{n^{14n}}}{-\dfrac{[(n+1)!]^2}{(n+1)^{14n+2}}} =$$

$$= -\lim_{n\to+\infty} \left(\frac{n!}{(n+1)!}\right)^2 \cdot \frac{(n+1)^{14n}\cdot(n+1)^2}{n^{14n}} =$$

$$= -\lim_{n\to+\infty} \left(\frac{\cancel{n!}}{(n+1)\,\cancel{n!}}\right)^2 \cdot \left(1+\frac{1}{n}\right)^{14n} \cdot (n+1)^2 =$$

$$= -\lim_{n\to+\infty} \frac{1}{\cancel{(n+1)^2}} \cdot \Bigg[\underbrace{\left(1+\frac{1}{n}\right)^n}_{e}\Bigg]^{14} \cdot \cancel{(n+1)^2} = -e^{14}$$

(16/01/2018)

Calcolare $\lim_{x\to 0} \dfrac{\sin x - \arctan x}{\left(e^{x/4} - 1 + \sinh(x^{10})\right)\left(\frac{1}{x}\log(1+x^3) + \cos x - 1\right)} = (\Diamond)$

NUMERATORE

$$\sin x = x - \frac{x^3}{6} + o(x^4)$$
$$\arctan x = x - \frac{x^3}{3} + o(x^4) \qquad \text{per } x \to 0$$
$$\sin x - \arctan x = -\frac{x^3}{6} + \frac{x^3}{3} + o(x^4)$$
$$= \frac{x^3}{6} + o(x^4) \sim \frac{x^3}{6} \quad \text{per } x \to 0$$

DENOMINATORE

$\sinh t = t + o(t)$ per $t \to 0$ $\qquad$ $e^t = 1 + t + o(t)$ per $t \to 0$

- $$e^{\frac{x}{4}} - 1 + \sinh(x^{10}) = \cancel{1} + \frac{x}{4} + o(x) - \cancel{1} + \underbrace{x^{10}}_{=o(x)} + \underbrace{o(x^{10})}_{=o(x)} =$$
$$= \frac{x}{4} + o(x) \sim \frac{x}{4} \quad \text{per } x \to 0$$

$$\log(1+x^3) = x^3 - \frac{x^6}{2} + o(x^6) \qquad \log(1+t) = t - \frac{t^2}{2} + o(t^2) \text{ per } t \to 0$$

$$\frac{1}{x}\log(1+x^3) = x^2 - \frac{x^5}{2} + o(x^5) \quad \text{per } x \to 0$$

- $$\frac{1}{x}\log(1+x^3) + \cos x - 1 = x^2 - \underbrace{\frac{x^5}{2}}_{=o(x^2)} + \underbrace{o(x^5)}_{=o(x^2)} + \cancel{1} - \frac{x^2}{2} + \underbrace{o(x^3)}_{=o(x^2)} - \cancel{1} =$$
$$= \frac{x^2}{2} + o(x^2) \sim \frac{x^2}{2} \quad \text{per } x \to 0$$

$$(\Diamond) = \lim_{x\to 0} \frac{\frac{\cancel{x^3}}{6}}{\frac{\cancel{x}}{4}\cdot\frac{\cancel{x^2}}{2}} = \frac{1}{6}\cdot 4\cdot 2 = \frac{4}{3}$$

Calcolare $\displaystyle\lim_{x\to 0} \frac{\cos(2x)+\sin^2 x - \frac{1}{1+x^2}}{e^{x^4}-1} = (\Delta)$

DENOMINATORE

$$e^{x^4}-1 \sim x^4 \quad \text{per } x\to 0$$

NUMERATORE

$$\frac{1}{1+t} = (1+t)^{-1} = 1-t+t^2-t^3+o(t^3) \quad \text{per } t\to 0$$

$$\frac{1}{1+x^2} = 1-x^2+x^4+o(x^4) \quad \text{per } x\to 0$$

$$\cos(2x) = 1-\frac{(2x)^2}{2}+\frac{(2x)^4}{4!}+o(x^5) \quad \text{per } x\to 0$$

$$\sin^2 x = (\sin x)^2 = \left(x-\frac{x^3}{6}+o(x^4)\right)^2 =$$

$$= x^2-\frac{1}{3}x^4+o(x^4) \quad \text{per } x\to 0$$

- $$\cos(2x)+\sin^2 x-\frac{1}{1+x^2} = 1-2x^2+\frac{16x^4}{24}+o(x^5)+$$

$$+\,x^2-\frac{1}{3}x^4+o(x^4)+$$

$$-\left(1-x^2+x^4+o(x^4)\right) =$$

$$= \cancel{1} - \cancel{2x^2} + \frac{2}{3}x^4 + \overbrace{o(x^5)}^{=o(x^4)} + \cancel{x^2} - \frac{1}{3}x^4 + o(x^4) - \cancel{1}$$

$$+ \cancel{x^2} - x^4 + o(x^4) =$$

$$= -\frac{2}{3}x^4 + o(x^4) \quad \text{per } x \to 0$$

$$(\Delta) = \lim_{x\to 0} \frac{-\frac{2}{3}x^4 + o(x^4)}{x^4 + o(x^4)} = -\frac{2}{3}$$

Compito

(25/06/2014) Calcolare, al variare di $\alpha < 0$,

$$\lim_{x\to+\infty} \frac{x\sin\left(\frac{1}{\sqrt{x}}\right) - \sqrt{x}}{\arctan\left(\frac{x^\alpha}{6}\right)}$$

DENOMINATORE

Se $\alpha < 0$, allora $\lim_{x\to+\infty} x^\alpha = 0$, quindi

$$\arctan\left(\frac{x^\alpha}{6}\right) \sim \frac{x^\alpha}{6} \quad \text{per } x \to +\infty$$

NUMERATORE

$$\sin\left(\frac{1}{\sqrt{x}}\right) = \frac{1}{\sqrt{x}} - \frac{1}{6}\cdot\left(\frac{1}{\sqrt{x}}\right)^3 + o\left(\left(\frac{1}{\sqrt{x}}\right)^4\right) \quad \text{per } x \to +\infty$$

$$= x^{-1/2} - \frac{1}{6}x^{-3/2} + o(x^{-2}) \quad \text{per } x \to +\infty$$

Esercizio da completare