Analisi Matematica 1 – Esercitazione del 21 novembre 2024

1. Calcolare il limite $\lim_{x\to 0} \frac{\sqrt[3]{1+x}+\sqrt[3]{1-x}-2}{1-\cos x+x^4}$.

2. Calcolare il limite $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\left(\frac{1}{\arctan x}\right)^2}$.

3. (Prova d'esame 3 luglio 2017) Calcolare il limite $\lim_{x\to 0} \frac{\log\left(\frac{1+x}{1+3x}\right)+2\sin x}{\cosh(3x)-\cos(3x)}$.

4. (Prova d'esame 3 febbraio 2020) Calcolare il limite $\lim_{x\to 0} \frac{\log(\cos(2x^2))}{\sqrt{1+2x^4}-\sqrt[3]{1+2x^4}}$.

5. (Prova d'esame 16 gennaio 2018) Calcolare il limite $\lim_{n\to+\infty} \frac{[e^{2n}-\left(\frac{1}{2}\right)^n][(n+1)!-n!]}{(n!-7^n)[\sin\frac{1}{n}-\frac{1}{4}\sin\frac{4}{n}]\sqrt{n^8e^{4n}+\sin\left(\frac{n}{7}\right)}}$.

6. (Prova d'esame 2 luglio 2019) Determinare per quali valori di $\alpha \in \mathbb{R}$ esiste finito il limite
$$\lim_{x\to 0^+} \frac{e^{x\cos x - x}-1}{(2x-\sin(2x))^{7\alpha}}.$$

7. (Prova d'esame 31 agosto 2015) Calcolare il limite
$$\lim_{n\to+\infty} \frac{\cosh\left(\frac{n!}{n^{7n}}\right)-\cos\left(\frac{n!}{n^{7n}}\right)}{1+\log\left(1+\frac{(n+1)!}{(n+1)^{7n+1}}\right)-\exp\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right)}.$$

8. (Test 16 gennaio 2018) Calcolare il limite
$$\lim_{x\to 0} \frac{\sin x-\arctan x}{(e^{x/4}-1+\sinh(x^{10}))\left(\frac{1}{x}\log(1+x^3)+\cos x-1\right)}.$$

9. Calcolare il limite $\lim_{x\to 0} \frac{\cos(2x)+\sin^2 x-\frac{1}{1+x^2}}{e^{x^4}-1}$.

10. (Prova d'esame 3 febbraio 2020) Sia $\alpha \in \mathbb{R}$. Calcolare il limite
$$\lim_{n\to+\infty} \frac{\left(1+\log\left(1+\frac{1}{n}\right)-\arctan\frac{1}{n}-\cosh\frac{1}{n}\right)^2}{e^{-n}+\frac{1}{3}\left(\frac{1}{n}\right)^{3\alpha}}.$$

Calcolare $\lim_{x\to 0} \dfrac{\sqrt[3]{1+x}+\sqrt[3]{1-x}-2}{1-\cos x+x^4} = \circledast$

$\sqrt[3]{1+x} \to 1$, $\sqrt[3]{1-x} \to 1$, $\cos x \to 1$, $x^4 \to 0$

$$(1+t)^{\alpha} = 1+\alpha t+\frac{\alpha(\alpha-1)}{2}t^2+o(t^2) \quad \text{per } t\to 0$$

$$\alpha = \frac{1}{3}$$

$t = x$

$$(1+x)^{1/3} = 1+\frac{1}{3}x+\frac{1}{2}\cdot\frac{1}{3}\cdot\left(-\frac{2}{3}\right)x^2+o(x^2) \quad \text{per } x\to 0$$

$t=-x$

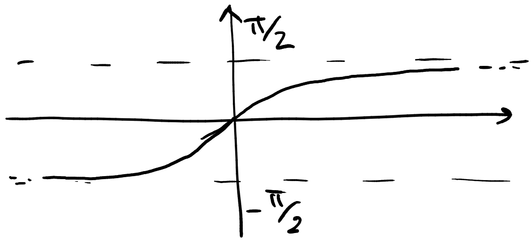
$$(1-x)^{1/3} = 1-\frac{1}{3}x+\frac{1}{2}\cdot\frac{1}{3}\cdot\left(-\frac{2}{3}\right)x^2+o(x^2) \quad \text{per } x\to 0$$

$$\sqrt[3]{1+x}+\sqrt[3]{1-x}-2 = \cancel{1}+\cancel{\frac{1}{3}x}-\frac{1}{9}x^2+o(x^2)+$$
$$+\cancel{1}-\cancel{\frac{1}{3}x}-\frac{1}{9}x^2+o(x^2)\cancel{-2} =$$
$$= -\frac{2}{9}x^2+o(x^2) \quad \text{per } x\to 0$$

$$1-\cos x+x^4 = 1-\left(1-\frac{x^2}{2}+o(x^3)\right)+x^4 =$$
$$= \frac{x^2}{2}+o(x^3)+\underbrace{x^4}_{=\,o(x^3)} = \frac{x^2}{2}+o(x^3) \quad \text{per } x\to 0$$

$$\circledast = \lim_{x\to 0}\frac{-\frac{2}{9}x^2+o(x^2)}{\frac{x^2}{2}+o(x^3)} = \lim_{x\to 0}\frac{-\frac{2}{9}\cancel{x^2}}{\frac{\cancel{x^2}}{2}} = -\frac{4}{9}$$

Calcolare $\displaystyle\lim_{x\to 0}\left(\underbrace{\frac{\sin x}{x}}_{\to 1}\right)^{\overbrace{\left(\frac{1}{\arctan x}\right)^2}^{\to +\infty}}$



$$\lim_{x\to 0}\arctan x = 0$$

$$\lim_{x\to 0}\frac{1}{\underbrace{\arctan^2 x}_{\to 0^+}} = +\infty$$

$\boxed{\text{NOTAZIONE} \quad \exp(t) = e^t \quad \forall t\in\mathbb{R}}$

$$\left(\frac{\sin x}{x}\right)^{\frac{1}{\arctan^2 x}} = \exp\left(\frac{1}{\arctan^2 x}\log\left(\frac{\sin x}{x}\right)\right)$$

$\arctan x \sim x$ per $x\to 0$

$\arctan^2 x \sim x^2$ per $x\to 0$

$$\sin x = x - \frac{x^3}{6} + o(x^4) \quad \text{per } x\to 0$$

$$\frac{\sin x}{x} = 1 - \frac{x^2}{6} + o(x^3) \quad \text{per } x\to 0$$

$\log(1+t) = t + o(t)$ per $t\to 0$

$$\log\left(\frac{\sin x}{x}\right) = \log\left(1 - \frac{x^2}{6} + o(x^3)\right) =$$

$$= -\frac{x^2}{6} + o(x^3) + o\left(-\frac{x^2}{6} + o(x^3)\right) =$$

$$= -\frac{x^2}{6} + o(x^2) \quad \text{per } x\to 0$$

$$\exp\left(\frac{1}{\arctan^2 x}\cdot\log\left(\frac{\sin x}{x}\right)\right)\sim\exp\left(\frac{1}{\cancel{x^2}}\cdot\left(-\frac{\cancel{x^2}}{6}\right)\right)=e^{-\frac{1}{6}}$$

per $x\to 0$

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^{\frac{1}{\arctan^2 x}}=e^{-1/6}$$

(03/07/2017)

Calcolare $\lim_{x\to 0} \dfrac{\log\left(\frac{1+x}{1+3x}\right)+2\sin x}{\cosh(3x)-\cos(3x)} = (\Delta)$

$$\text{NUMERATORE} = \log\left(\frac{1+x}{1+3x}\right)+2\sin x = \log(1+x)-\log(1+3x)+2\sin x$$

$$\log(1+x) = x-\frac{x^2}{2}+o(x^2) \quad \text{per } x\to 0$$

$$\sin x = x+o(x^2) \quad \text{per } x\to 0$$

$$\log(1+3x) = 3x-\frac{(3x)^2}{2}+o(x^2) \quad \text{per } x\to 0$$

$$\text{NUMERATORE} = \cancel{x}-\frac{x^2}{2}+o(x^2)-\cancel{3x}+\frac{9}{2}x^2+o(x^2)+\cancel{2x}+o(x^2) =$$

$$= 4x^2+o(x^2) \quad \text{per } x\to 0$$

DENOMINATORE

$$\cosh(3x) = 1+\frac{(3x)^2}{2}+o(x^3) \quad \text{per } x\to 0$$

$$\cos(3x) = 1-\frac{(3x)^2}{2}+o(x^3) \quad \text{per } x\to 0$$

$$\cosh(3x)-\cos(3x) = \cancel{1}+\frac{9}{2}x^2+o(x^3)-\cancel{1}+\frac{9}{2}x^2+o(x^3) =$$

$$= 9x^2+o(x^3) \quad \text{per } x\to 0$$

$$(\Delta) = \lim_{x\to 0}\frac{4x^2+o(x^2)}{9x^2+o(x^3)} = \lim_{x\to 0}\frac{4\cancel{x^2}}{9\cancel{x^2}} = \frac{4}{9}$$

(03/02/2020) Calcolare

$$\lim_{x \to 0} \frac{\log(\cos(2x^2))}{\sqrt{1+2x^4} - \sqrt[3]{1+2x^4}} = (\square)$$

$\cos t = 1 - \frac{t^2}{2} + o(t^3)$ per $t \to 0$

NUMERATORE

$$\cos(2x^2) = 1 - \frac{(2x^2)^2}{2} + o(x^6) =$$

$$= 1 - 2x^4 + o(x^6)$$

$$\log(\cos(2x^2)) = \log(1 - 2x^4 + o(x^6)) =$$

$$= -2x^4 + o(x^6) + o(-2x^4 + o(x^6)) =$$

$$= -2x^4 + o(x^4) \quad \text{per } x \to 0$$

$(1+t)^\alpha = 1 + \alpha t + o(t)$ per $t \to 0$

DENOMINATORE

$$(1+2x^4)^{1/2} - (1+2x^4)^{1/3} =$$

$$= 1 + \frac{1}{2} \cdot 2x^4 + o(x^4) - \left(1 + \frac{1}{3} \cdot 2x^4 + o(x^4)\right) =$$

$$= x^4 + o(x^4) - \frac{2}{3}x^4 + o(x^4) = \frac{1}{3}x^4 + o(x^4) \quad \text{per } x \to 0$$

$$(\square) = \lim_{x \to 0} \frac{-2x^4 + o(x^4)}{\frac{1}{3}x^4 + o(x^4)} = \lim_{x \to 0} \frac{-2\cancel{x^4}}{\frac{1}{3}\cancel{x^4}} = -6$$

(16/01/2018) Calcolare

$$\lim_{n\to+\infty} \frac{\left[e^{2n}-\left(\frac{1}{2}\right)^n\right]\left[(n+1)!-n!\right]}{(n!-7^n)\Big[\underbrace{\sin\left(\frac{1}{n}\right)}_{\downarrow\, 0} - \frac{1}{4}\underbrace{\sin\left(\frac{4}{n}\right)}_{\downarrow\, 0}\Big]\sqrt{n^8 e^{4n}+\sin\left(\frac{n}{7}\right)}} = \circledast$$

NUMERATORE

- $\underbrace{e^{2n}}_{\downarrow\, +\infty} - \underbrace{\left(\frac{1}{2}\right)^n}_{\downarrow\, 0} \sim e^{2n}$ per $n\to+\infty$

- $(n+1)!-n! = (n+1)!\Big[1-\underbrace{\frac{\cancel{n!}}{(n+1)\cancel{n!}}}_{\downarrow\, 0}\Big] \sim (n+1)!$ per $n\to+\infty$

DENOMINATORE

- $n!-7^n \sim n!$ per $n\to+\infty$

  Confronto di infiniti $\lim_{n\to+\infty}\frac{7^n}{n!}=0$

- $\sin\left(\frac{1}{n}\right)-\frac{1}{4}\sin\left(\frac{4}{n}\right)$

$$\boxed{\sin t = t-\frac{t^3}{6}+o(t^4) \quad \text{per } t\to 0}$$

$$= \frac{1}{n}-\frac{1}{6}\cdot\left(\frac{1}{n}\right)^3+o\left(\frac{1}{n^4}\right)-\frac{1}{4}\left[\frac{4}{n}-\frac{1}{6}\cdot\left(\frac{4}{n}\right)^3+o\left(\frac{1}{n^4}\right)\right]=$$

$$= \cancel{\frac{1}{n}}-\frac{1}{6n^3}+o\left(\frac{1}{n^4}\right)-\cancel{\frac{1}{n}}+\frac{16}{6n^3}+o\left(\frac{1}{n^4}\right)=$$

$$= \frac{\cancel{15}^{\,5}}{{}_2\cancel{6}\, n^3} + o\left(\frac{1}{n^4}\right) = \frac{5}{2n^3} + o\left(\frac{1}{n^4}\right) \quad \text{per } n \to +\infty$$

$$\bullet \sqrt{n^8 e^{4n} + \sin\left(\frac{n}{7}\right)} = \sqrt{n^8 e^{4n}\left(1 + \underbrace{\frac{\sin(n/7)}{n^8 e^{4n}}}_{\to 0}\right)} \sim \sqrt{n^8 e^{4n}} = n^4 e^{2n}$$

per $n \to +\infty$

0 (succ. infinitesima per succ. limitata)

$$\circledast = \lim_{n\to+\infty} \frac{\cancel{e^{2n}} \cdot (n+1)!}{n! \cdot \frac{5}{2\cancel{n^3}_{\,1}} \cdot \cancel{n^4}^{\,n}\, \cancel{e^{2n}}} =$$

$$= \lim_{n\to+\infty} \frac{2}{5} \cdot \frac{(n+1)!}{n \cdot n!} =$$

$$= \lim_{n\to+\infty} \frac{2}{5} \cdot \frac{(n+1)\cancel{n!}}{n \cdot \cancel{n!}} = \lim_{n\to+\infty} \frac{2}{5} \cdot \underbrace{\frac{n+1}{n}}_{\to 1} = \frac{2}{5}$$

$\sim n$

(02/07/2019) Determinare per quali $\alpha \in \mathbb{R}$ esiste finito $\lim_{x\to 0} \dfrac{e^{x\cos x - x} - 1}{(2x-\sin(2x))^{7\alpha}} = (\Delta)$

NUMERATORE

$$x\cos x - x = x(\cos x - 1) =$$
$$= x \cdot \left(\cancel{1} - \frac{x^2}{2} + o(x^3) - \cancel{1}\right) =$$
$$= -\frac{x^3}{2} + o(x^4) \quad \text{per } x \to 0$$

$$e^t = 1 + t + o(t) \quad \text{per } t \to 0$$

$$e^{x\cos x - x} - 1 = e^{-\frac{x^3}{2} + o(x^4)} - 1 =$$
$$= -\frac{x^3}{2} + o(x^4) + o\left(-\frac{x^3}{2} + o(x^4)\right) =$$
$$= -\frac{x^3}{2} + o(x^3) \quad \text{per } x \to 0$$

DENOMINATORE

$$2x - \sin(2x) = 2x - \left(2x - \frac{(2x)^3}{6} + o(x^4)\right) =$$
$$= \frac{8}{6}x^3 + o(x^4) = \frac{4}{3}x^3 + o(x^4) \quad \text{per } x \to 0$$

$$(2x - \sin(2x))^{7\alpha} \sim \left(\frac{4}{3}x^3\right)^{7\alpha} = \left(\frac{4}{3}\right)^{7\alpha} x^{21\alpha}$$

$$(\triangle) = \lim_{x\to 0} \frac{-\frac{x^3}{2}}{\left(\frac{4}{3}\right)^{7\alpha} x^{21\alpha}} =$$

$$= \lim_{x\to 0} \left(-\frac{1}{2}\left(\frac{3}{4}\right)^{7\alpha}\right) x^{3-21\alpha}$$

Il limite è finito se e solo se $3-21\alpha \geq 0$,

cioè $\alpha \leq \frac{1}{7}$.

Esponente $3-21\alpha$ positivo

$$(*) = \lim_{x\to 0} \left(-\frac{1}{2}\left(\frac{3}{4}\right)^{7\alpha}\right) x^{3-21\alpha} = \begin{cases} 0 & \text{se } \alpha < \frac{1}{7} \\ -\frac{1}{2}\cdot\frac{3}{4} = -\frac{3}{8} & \text{se } \alpha = \frac{1}{7} \end{cases}$$

[Se $\alpha > \frac{1}{7}$, allora $\lim_{x\to 0^+} \left(-\frac{1}{2}\left(\frac{3}{4}\right)^{7\alpha}\right) x^{3-21\alpha} = +\infty$

Se $x<0$, allora $x^{3-21\alpha}$ non è definita per tutti gli $\alpha > \frac{1}{7}$.]

(31/08/2015) Calcolare

$$\lim_{n\to+\infty} \frac{\cosh\left(\frac{n!}{n^{7n}}\right)-\cos\left(\frac{n!}{n^{7n}}\right)}{1+\log\left(1+\frac{(n+1)!}{(n+1)^{7n+1}}\right)-\exp\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right)} = (\square)$$

$\lim_{n\to+\infty} \frac{n!}{n^{7n}} = 0$ e $\lim_{n\to+\infty} \frac{(n+1)!}{(n+1)^{7n+1}} = 0$ per confronto di infiniti

NUMERATORE

- $\cosh(t) - \cos(t) =$

$$= 1 + \frac{t^2}{2} + o(t^3) - \left(1 - \frac{t^2}{2} + o(t^3)\right) = t^2 + o(t^3) \quad \text{per } t\to 0$$

$$\cosh\left(\frac{n!}{n^{7n}}\right) - \cos\left(\frac{n!}{n^{7n}}\right) \sim \left(\frac{n!}{n^{7n}}\right)^2 \quad \text{per } n\to+\infty$$

DENOMINATORE

$$1 + \log(1+t) - e^t =$$

$$= 1 + \left(t - \frac{t^2}{2} + o(t^2)\right) - \left(1 + t + \frac{t^2}{2} + o(t^2)\right) =$$

$$= \cancel{1} + \cancel{t} - \frac{t^2}{2} + o(t^2) - \cancel{1} - \cancel{t} - \frac{t^2}{2} + o(t^2) =$$

$$= -t^2 + o(t^2) \quad \text{per } t\to 0$$

$$1 + \log\left(1+\frac{(n+1)!}{(n+1)^{7n+1}}\right) - \exp\left(\frac{(n+1)!}{(n+1)^{7n+1}}\right) \sim -\left[\frac{(n+1)!}{(n+1)^{7n+1}}\right]^2$$

per $n\to+\infty$

$$(\square) = \lim_{n\to+\infty} \frac{\left(\dfrac{n!}{n^{7n}}\right)^2}{-\left[\dfrac{(n+1)!}{(n+1)^{7n+1}}\right]^2} =$$

$$= -\lim_{n\to+\infty} \left(\frac{n!}{n^{7n}} \cdot \frac{(n+1)^{7n+1}}{(n+1)!}\right)^2 =$$

$$= -\lim_{n\to+\infty} \left(\frac{\cancel{n!}}{n^{7n}} \cdot \frac{(n+1)^{7n}\cdot\cancel{(n+1)}}{\cancel{(n+1)}\cdot\cancel{n!}}\right)^2 =$$

$$= -\lim_{n\to+\infty} \left(\left(\frac{n+1}{n}\right)^{7n}\right)^2 =$$

$$= -\lim_{n\to+\infty} \left(\left(1+\frac{1}{n}\right)^{n}\right)^{14} = -e^{14}$$

(Test 16/01/2018) Calcolare

$$\lim_{x\to 0} \frac{\overset{\nearrow 0}{\sin x} - \overset{\nearrow 0}{\arctan x}}{\left(\underset{\downarrow\, 1}{e^{\frac{x}{4}}} - 1 + \underset{\downarrow\, 0}{\sinh(x^{10})}\right)\left(\frac{1}{x}\log(1+x^3) + \underset{\downarrow\, 1}{\cos x} - 1\right)} = \circledast$$

NUMERATORE

- $\sin x - \arctan x = x - \frac{x^3}{6} + o(x^4) - \left(x - \frac{x^3}{3} + o(x^4)\right) =$

$$= x - \frac{x^3}{6} + o(x^4) - x + \frac{x^3}{3} + o(x^4) =$$

$$= \frac{x^3}{6} + o(x^4) \quad \text{per } x \to 0$$

DENOMINATORE

$\sinh(t) = t + o(t)$ per $t \to 0$

- $e^{\frac{x}{4}} - 1 + \sinh(x^{10}) = \cancel{1} + \frac{x}{4} + o(x) - \cancel{1} + x^{10} + o(x^{10}) =$

$$= \frac{x}{4} + o(x) \quad \text{per } x \to 0$$

- $\log(1 + x^3) = x^3 - \frac{x^6}{2} + o(x^6)$

$\log(1+t) = t - \frac{t^2}{2} + o(t^2)$

$$\frac{1}{x}\log(1+x^3) = x^2 - \frac{x^5}{2} + o(x^5) \quad \text{per } x \to 0$$

Dal seguito si vede questo termine dello sviluppo non serve

$$\frac{1}{x}\log(1+x^3) + \cos x - 1 = x^2 - \frac{x^5}{2} + o(x^5) + \cancel{1} - \frac{x^2}{2} + o(x^3) - \cancel{1} =$$

$$= \frac{1}{2}x^2 + o(x^2)$$

$$\circledast = \lim_{x \to 0} \frac{\frac{x^3}{6} + o(x^4)}{\left(\frac{x}{4} + o(x)\right) \cdot \left(\frac{x^2}{2} + o(x^2)\right)} =$$

$$= \lim_{x \to 0} \frac{\frac{\cancel{x^3}}{6}}{\frac{\cancel{x}}{4} \cdot \frac{\cancel{x^2}}{2}} = \frac{1}{6} \cdot 8 = \frac{4}{3}$$

Calcolare $\lim_{x\to 0} \dfrac{\cos(2x)+\sin^2 x - \frac{1}{1+x^2}}{e^{x^4}-1} = (\Delta)$

• DENOMINATORE

$$e^{x^4}-1 \sim x^4 \quad \text{per } x\to 0$$

• NUMERATORE

$$\frac{1}{1+t} = (1+t)^{-1} = 1 - t + t^2 - t^3 + t^4 + o(t^4)$$

$$\frac{1}{1+x^2} = 1 - x^2 + x^4 + o(x^4)$$

$$\cos(2x) = 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24} + o(x^5)$$

$$\sin x = x - \frac{x^3}{6} + o(x^4)$$

$$(\sin x)^2 = \left(x - \frac{x^3}{6} + o(x^4)\right)^2 = x^2 - \frac{1}{3}x^4 + o(x^4)$$

$$\cos(2x) + \sin^2 x - \frac{1}{1+x^2} =$$

$$= \cancel{1} - \cancel{2x^2} + \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{24}}}x^4 + \cancel{x^2} - \frac{1}{3}x^4 + o(x^4) - \cancel{1} + \cancel{x^2} - x^4 + o(x^4) =$$

$$= -\frac{2}{3}x^4 + o(x^4) \quad \text{per } x \to 0$$

$$(\Delta) = \lim_{x \to 0} \frac{-\frac{2}{3}x^4}{x^4} = -\frac{2}{3}$$

(03/02/2020) Calcolare, al variare di $\alpha \in \mathbb{R}$,

$$\lim_{n\to+\infty} \frac{\left(1+\log\left(1+\frac{1}{n}\right)-\arctan\frac{1}{n}-\cosh\frac{1}{n}\right)^2}{e^{-n}+\frac{1}{3}\left(\frac{1}{n}\right)^{3\alpha}} = \circledast$$

NUMERATORE

$$1+\log\left(1+\frac{1}{n}\right)-\arctan\frac{1}{n}-\cosh\frac{1}{n} =$$

$$= \cancel{1} + \cancel{\frac{1}{n}} - \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right) - \cancel{\frac{1}{n}} + o\left(\frac{1}{n^2}\right) - \cancel{1} - \frac{1}{2n^2} + o\left(\frac{1}{n^2}\right) =$$

$$= -\frac{1}{n^2} + o\left(\frac{1}{n^2}\right) \quad \text{per } n\to+\infty$$

Il numeratore si comporta come $\left(-\frac{1}{n^2}\right)^2 = \frac{1}{n^4}$ per $n\to+\infty$

$$\circledast = \lim_{n\to+\infty} \frac{\frac{1}{n^4}}{\frac{1}{e^n}+\frac{1}{3}n^{-3\alpha}} =$$

$$= \lim_{n\to+\infty} \frac{1}{n^4} \cdot \frac{1}{\frac{1}{e^n}+\frac{1}{3}n^{-3\alpha}} =$$

$$= \lim_{n\to+\infty} \frac{1}{\underbrace{\frac{n^4}{e^n}}_{0}+\frac{1}{3}n^{4-3\alpha}}$$

per confronti di infiniti

$4-3\alpha>0 \iff \alpha<\frac{4}{3}$

$$\bullet \lim_{n\to+\infty} n^{4-3\alpha} = \begin{cases} +\infty & \text{se } \alpha<\frac{4}{3} \\ 1 & \text{se } \alpha=\frac{4}{3} \\ 0 & \text{se } \alpha>\frac{4}{3} \end{cases}$$

$$\circledast = \lim_{n\to+\infty} \frac{1}{\frac{n^4}{e^n}+\frac{1}{3}n^{4-3\alpha}} =$$

$$= \begin{cases} \frac{1}{0+\infty}=0 & \text{se } \alpha<\frac{4}{3} \\ \frac{1}{0+\frac{1}{3}\cdot 1}=3 & \text{se } \alpha=\frac{4}{3} \\ +\infty & \text{se } \alpha>\frac{4}{3} \end{cases}$$