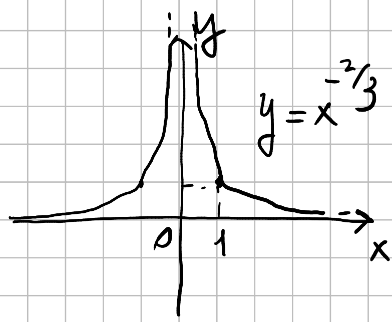
Analisi Matematica 1 – Esercitazione del 17 ottobre 2024

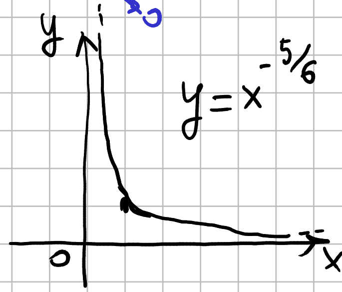
1. Calcolare il limite $\lim_{x\to+\infty} \dfrac{2x^{2/3} - 5x^{4/3} + \sqrt{x}}{3x + 6x^{2/3} - 2x\sqrt[3]{x}}$.
2. Calcolare i limiti $\lim_{x\to+\infty} \dfrac{\arctan x}{x}$ e $\lim_{x\to-\infty} \dfrac{\arctan x}{x}$.
3. Calcolare i limiti $\lim_{x\to0^+} \dfrac{\cos x}{x}$, $\lim_{x\to0^-} \dfrac{\cos x}{x}$ e $\lim_{x\to0} \dfrac{\cos x}{x^2}$.
4. Calcolare i limiti $\lim_{x\to0^+} \dfrac{\arccos x}{x}$ e $\lim_{x\to0^-} \dfrac{\arccos x}{x}$.
5. Calcolare il limite $\lim_{x\to+\infty} \dfrac{\log x - 2\arctan x}{3\sqrt[4]{x} + 6x^{1/\sqrt{2}} - \log x}$.
6. Calcolare il limite $\lim_{x\to+\infty} \dfrac{e}{\sqrt{2x} - \sqrt{2x+3}}$.
7. Calcolare il limite $\lim_{x\to0} \dfrac{\sqrt{1-3x} - \sqrt{1-x}}{8x}$.
8. Calcolare il limite $\lim_{x\to+\infty} \dfrac{\left(3 + \dfrac{3}{x}\right)^x}{e^x - \left(\frac{3}{2}\right)^x}$.
9. Calcolare il limite $\lim_{x\to+\infty} \dfrac{x^x}{(x+1)^{x+3}}$.
10. Calcolare il limite $\lim_{x\to0} \dfrac{\sin x + x^2}{3\sin x - x}$.
11. Calcolare il limite $\lim_{x\to0^+} \dfrac{x + \sqrt{|x|}}{\sin x}$ e $\lim_{x\to0^-} \dfrac{x + \sqrt{|x|}}{\sin x}$.
12. Calcolare il limite $\lim_{x\to0} \dfrac{\tan x}{x}$.
13. Calcolare il limite $\lim_{x\to0} \dfrac{1 - \cos x}{x^2}$.
14. Calcolare il limite $\lim_{x\to0} \dfrac{x\tan x}{1 - \cos x}$.
15. Calcolare il limite $\lim_{x\to0} \dfrac{\sin x - \tan x}{x^3}$.
16. Calcolare il limite $\lim_{x\to0} x \sin\dfrac{2}{x}$.
17. Calcolare il limite $\lim_{x\to-\infty} \dfrac{4x - 9x^{3/4}}{5x + 6\sin x}$.

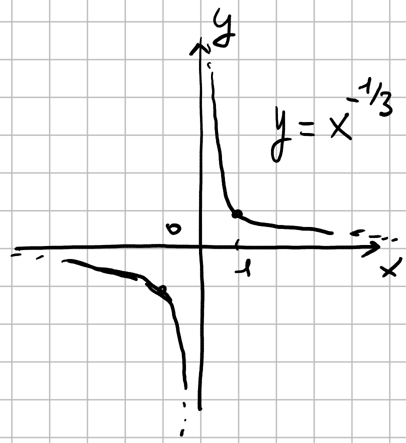
Calcolare $\lim_{x \to +\infty} \dfrac{2x^{2/3} - 5x^{4/3} + \sqrt{x}}{3x + 6x^{2/3} - 2x\sqrt[3]{x}}$

$$\lim_{x \to +\infty} \frac{2x^{2/3} - 5x^{4/3} + x^{1/2}}{3x + 6x^{2/3} - 2x^{4/3}} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^{4/3}}\left(2\underbrace{x^{-2/3}}_{\to 0} - 5 + \underbrace{x^{-\frac{5}{6}}}_{\to 0}\right)}{\cancel{x^{4/3}}\left(3\underbrace{x^{-1/3}}_{\to 0} + 6\underbrace{x^{-2/3}}_{\to 0} - 2\right)} = \frac{5}{2}$$





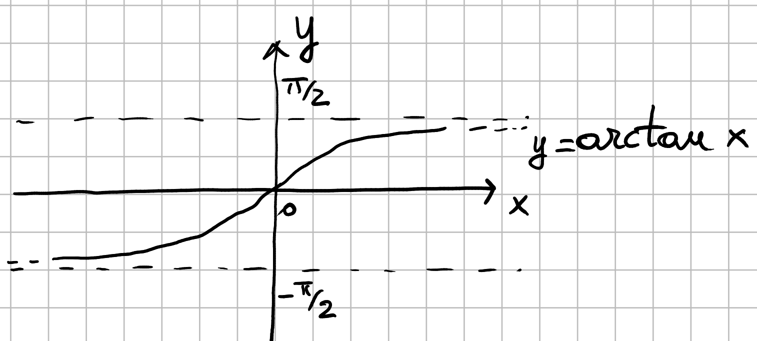


Abbiamo mostrato che

$$\left(2x^{2/3} - 5x^{4/3} + x^{1/2}\right) \sim -5x^{4/3} \quad \text{per } x \to +\infty \quad \text{e}$$

$$\left(3x + 6x^{2/3} - 2x^{4/3}\right) \sim -2x^{4/3} \quad \text{per } x \to +\infty$$

Calcolare $\lim_{x \to +\infty} \dfrac{\arctan x}{x}$ e $\lim_{x \to -\infty} \dfrac{\arctan x}{x}$

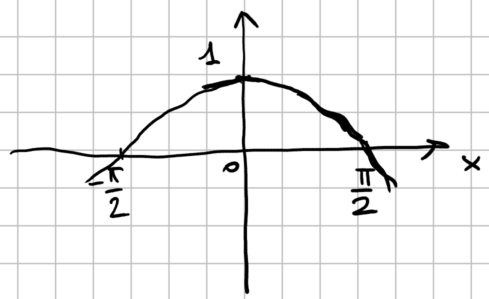


$$\lim_{x \to +\infty} \frac{\overbrace{\arctan x}^{\to \frac{\pi}{2}}}{\underbrace{x}_{\to +\infty}} = 0 \qquad \lim_{x \to -\infty} \frac{\overbrace{\arctan x}^{\to -\frac{\pi}{2}}}{\underbrace{x}_{\to -\infty}} = 0$$

algebra dei limiti

Osservazione. $f(x) = \frac{\arctan x}{x}$ è una funzione pari, quindi dal calcolo di $\lim_{x\to+\infty} \frac{\arctan x}{x}$, si può concludere direttamente che $\lim_{x\to-\infty} \frac{\arctan x}{x}$ è uguale al primo limite, o viceversa

Calcolare $\lim_{x\to0^+} \frac{\cos x}{x}$, $\lim_{x\to0^-} \frac{\cos x}{x}$ e $\lim_{x\to0} \frac{\cos x}{x^2}$



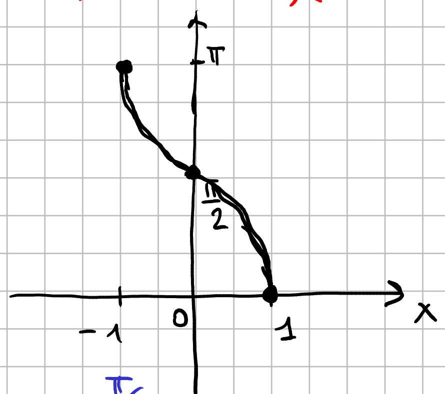
$$\lim_{x\to0} \cos x = 1$$

$$\lim_{x\to0^+} \frac{\overset{\nearrow 1}{\cos x}}{\underset{\searrow 0^+}{x}} \overset{AL}{=} +\infty \qquad \lim_{x\to0^-} \frac{\overset{\nearrow 1}{\cos x}}{\underset{\searrow 0^-}{x}} \overset{AL}{=} -\infty$$

Non esiste $\lim_{x\to0} \frac{\cos x}{x}$

$$\lim_{x\to0} \frac{\overset{\nearrow 1}{\cos x}}{\underset{\searrow 0^+}{x^2}} \overset{AL}{=} +\infty$$

Calcolare $\lim_{x\to0^+} \frac{\arccos x}{x}$ e $\lim_{x\to0^-} \frac{\arccos x}{x}$

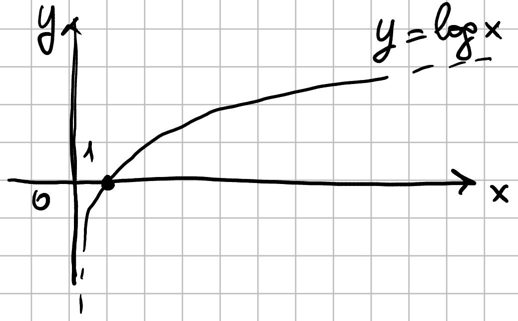


$$\lim_{x\to0} \arccos x = \frac{\pi}{2}$$

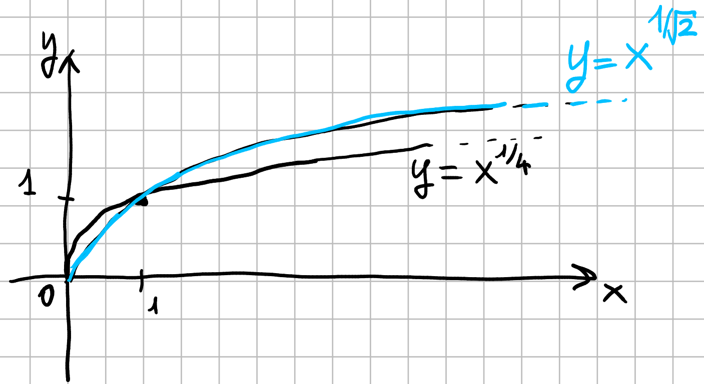
$$\lim_{x\to0^+} \frac{\overset{\nearrow \pi/2}{\arccos x}}{\underset{\searrow 0^+}{x}} = +\infty, \qquad \lim_{x\to0^-} \frac{\overset{\nearrow \frac{\pi}{2}}{\arccos x}}{\underset{\searrow 0^-}{x}} = -\infty$$

Calcolare $\lim\limits_{x\to+\infty} \dfrac{\log x - 2\arctan x}{3\sqrt[4]{x}+6x^{1/\sqrt{2}}-\log x}$

$$\log x - 2\arctan x = \underbrace{\log x}_{\to+\infty}\underbrace{\left(1-\frac{2\overbrace{\arctan x}^{\to\frac{\pi}{2}}}{\underbrace{\log x}_{\to+\infty}}\right)}_{\to 1} \sim \log x \quad \text{per } x\to+\infty$$



$$3x^{1/4}+6x^{1/\sqrt{2}}-\log x = x^{1/\sqrt{2}}\overbrace{\left(3\underbrace{x^{1/4-1/\sqrt{2}}}_{\to 0}+6-\underbrace{\frac{\log x}{x^{1/\sqrt{2}}}}_{\to 0}\right)}^{\to 6}$$

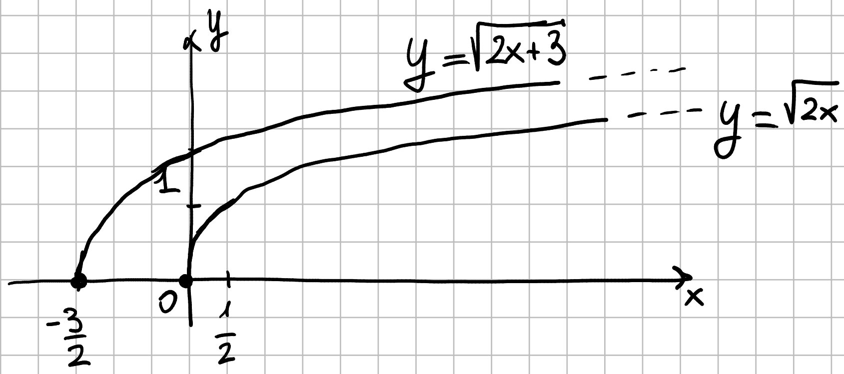


$$\boxed{\frac{\log x}{x^{\alpha}} \longrightarrow 0 \quad \text{per } x\to+\infty \quad \text{per ogni } \alpha>0}$$

$$\left(3x^{1/4}+6x^{1/\sqrt{2}}-\log x\right)\sim 6x^{1/\sqrt{2}} \quad \text{per } x\to+\infty$$

$$\lim_{x\to+\infty}\frac{\log x-2\arctan x}{3x^{1/4}+6x^{1/\sqrt{2}}-\log x} = \lim_{x\to+\infty}\frac{\log x}{6x^{1/\sqrt{2}}} = 0$$

Calcolare $\lim\limits_{x\to+\infty} \dfrac{e}{\sqrt{2x}-\sqrt{2x+3}}$

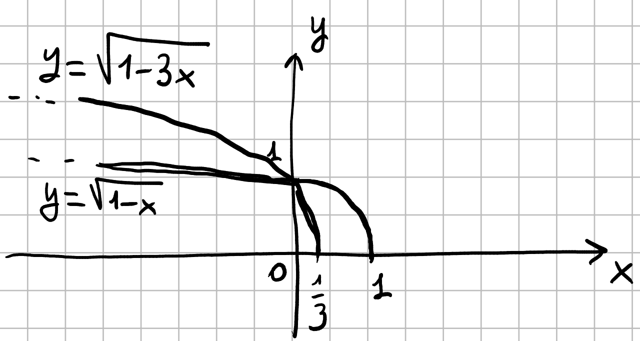


$$(A-B)\cdot(A+B) = A^2-B^2$$

$$\lim_{x\to+\infty} \frac{e}{\sqrt{2x}-\sqrt{2x+3}} = \lim_{x\to+\infty} \frac{e}{\sqrt{2x}-\sqrt{2x+3}}\cdot\frac{\sqrt{2x}+\sqrt{2x+3}}{\sqrt{2x}+\sqrt{2x+3}} =$$

$$= \lim_{x\to+\infty} \frac{e\overbrace{\left(\sqrt{2x}+\sqrt{2x+3}\right)}^{\to+\infty}}{\cancel{2x}-\cancel{2x}-3} \overset{AL}{=} -\infty$$

Calcolare $\lim\limits_{x\to 0} \dfrac{\sqrt{1-3x}-\sqrt{1-x}}{8x}$



$$\lim_{x\to 0}\sqrt{1-3x} = 1$$

$$\lim_{x\to 0}\sqrt{1-x} = 1$$

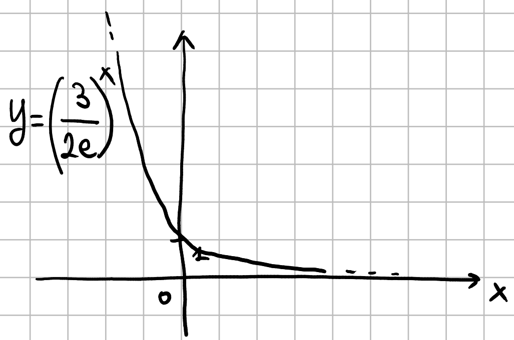
$$\lim_{x\to 0} \frac{\overbrace{\sqrt{1-3x}-\sqrt{1-x}}^{\to 0}}{\underbrace{8x}_{\to 0}} = \lim_{x\to 0} \frac{\sqrt{1-3x}-\sqrt{1-x}}{8x}\cdot\frac{\sqrt{1-3x}+\sqrt{1-x}}{\sqrt{1-3x}+\sqrt{1-x}} =$$

$$= \lim_{x\to 0} \frac{\cancel{1}-3x-(\cancel{1}-x)}{8x\left(\sqrt{1-3x}+\sqrt{1-x}\right)} = \lim_{x\to 0} \frac{-\overset{1}{\cancel{2}}\cancel{x}}{\underset{4}{\cancel{8}}\cancel{x}\left(\sqrt{1-3x}+\sqrt{1-x}\right)} =$$

$$= \lim_{x\to 0} \frac{-1}{4\left(\underbrace{\sqrt{1-3x}}_{\downarrow\, 1}+\underbrace{\sqrt{1-x}}_{\downarrow\, 1}\right)} \overset{AL}{=} \frac{-1}{4(1+1)} = -\frac{1}{8}$$

Calcolare $\displaystyle \lim_{x\to+\infty} \frac{\left(3+\frac{3}{x}\right)^x}{e^x-\left(\frac{3}{2}\right)^x}$

$$e^x - \left(\frac{3}{2}\right)^x = e^x\left(1-\underbrace{\left(\frac{3}{2e}\right)^x}_{\downarrow\, 0}\right) \sim e^x \quad \text{per } x\to+\infty$$

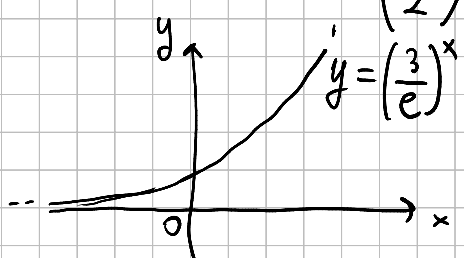


$$\left(3+\frac{3}{x}\right)^x = \left[3\left(1+\frac{1}{x}\right)\right]^x =$$

$$\boxed{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x = e}$$

$$= 3^x \underbrace{\left(1+\frac{1}{x}\right)^x}_{\downarrow\, e} \sim e\cdot 3^x \quad \text{per } x\to+\infty$$

$$\lim_{x\to+\infty} \frac{\left(3+\frac{3}{x}\right)^x}{e^x-\left(\frac{3}{2}\right)^x} = \lim_{x\to+\infty} \frac{e\cdot 3^x}{e^x} = \lim_{x\to+\infty} e\cdot\left(\frac{3}{e}\right)^x \underset{AL}{=} +\infty$$



Calcolare $\lim\limits_{x\to+\infty} \dfrac{x^x}{(x+1)^{x+3}}$

$$\lim_{x\to+\infty} \frac{x^x}{(x+1)^{x+3}} = \lim_{x\to+\infty} \frac{1}{(x+1)^3}\cdot\frac{x^x}{(x+1)^x} =$$

$$= \lim_{x\to+\infty} \frac{1}{(x+1)^3}\cdot\frac{1}{\frac{(x+1)^x}{x^x}} = \lim_{x\to+\infty} \underbrace{\frac{1}{(x+1)^3}}_{(x+1)^3\to+\infty}\cdot\underbrace{\frac{1}{\left(1+\frac{1}{x}\right)^x}}_{\left(1+\frac{1}{x}\right)^x\to e} \overset{AL}{=}$$

$$= 0\cdot\frac{1}{e} = 0$$

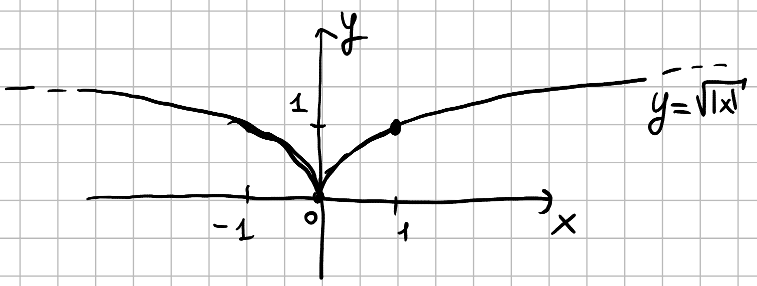
Calcolare $\lim\limits_{x\to 0} \dfrac{\sin x + x^2}{3\sin x - x}$

$$\lim_{x\to 0} \frac{\underbrace{\sin x}_{\to 0} + \underbrace{x^2}_{\to 0}}{3\sin x - \underbrace{x}_{\to 0}} =$$

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x} = 1}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\left(\frac{\sin x}{x}+x\right)}{\cancel{x}\left(3\frac{\sin x}{x}-1\right)} = \lim_{x\to 0} \frac{\underbrace{\frac{\sin x}{x}}_{\to 1}+\underbrace{x}_{\to 0}}{3\underbrace{\frac{\sin x}{x}}_{\to 1}-1} \overset{AL}{=} \frac{1+0}{3-1} = \frac{1}{2}$$

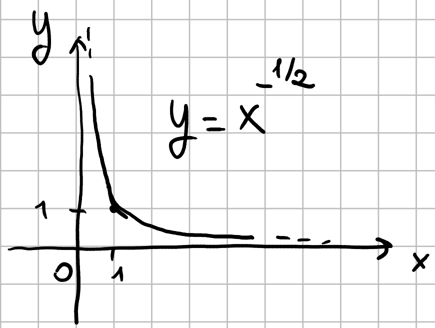
Calcolare $\lim\limits_{x\to 0^+} \dfrac{x+\sqrt{|x|}}{\sin x}$ e $\lim\limits_{x\to 0^-} \dfrac{x+\sqrt{|x|}}{\sin x}$



$$\lim_{x\to 0} \sqrt{|x|} = 0$$

$$\lim_{x\to 0^+} \frac{x+\sqrt{|x|}}{\sin x} = \lim_{x\to 0^+} \frac{\cancel{x}\left(1+\frac{1}{\sqrt{x}}\right)}{\cancel{x}\cdot\frac{\sin x}{x}} = \lim_{x\to 0^+} \frac{1+\overbrace{\frac{1}{\sqrt{x}}}^{+\infty}}{\underbrace{\frac{\sin x}{x}}_{1}} = +\infty$$

$\sqrt{|x|} = \sqrt{x}$ se $x>0$



$$\lim_{x\to 0^-} \frac{x+\sqrt{|x|}}{\sin x} = \lim_{x\to 0^-} \frac{\cancel{x}\left(1-\frac{1}{\sqrt{-x}}\right)}{\cancel{x}\cdot\frac{\sin x}{x}} = \lim_{x\to 0^-} \frac{1-\overbrace{\frac{1}{\sqrt{-x}}}^{+\infty}}{\underbrace{\frac{\sin x}{x}}_{1}} = -\infty$$

$\sqrt{|x|} = \sqrt{-x}$ se $x<0$

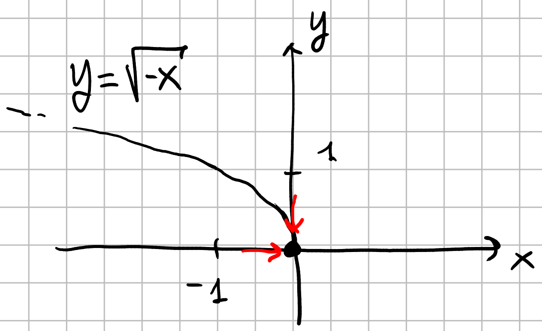
Spiegazione ulteriore dei passaggi algebrici precedenti

- Se $x>0$, allora

$$x+\sqrt{|x|} = x+\sqrt{x} = x\left(\frac{x}{x}+\frac{\sqrt{x}}{x}\right) = x\left(1+\frac{\sqrt{x}}{(\sqrt{x})^2}\right) = x\left(1+\frac{1}{\sqrt{x}}\right)$$

Alternativa

$$x+\sqrt{|x|} = x+x^{1/2} = x\left(\frac{x}{x}+\frac{x^{1/2}}{x}\right) = x\left(1+x^{\frac{1}{2}-1}\right) = x\left(1+x^{-\frac{1}{2}}\right)$$



$$\lim_{x\to 0^-}\sqrt{-x}=0$$

La funzione $y=\sqrt{-x}$ assume solo valori non negativi.

• Se $x<0$, allora

$$x+\sqrt{|x|}=x+\sqrt{-x}=x\left(1+\frac{\sqrt{-x}}{x}\right)=$$

$$=x\left(1+\frac{\sqrt{-x}}{-(-x)}\right)=$$

$$=x\left(1-\frac{\sqrt{-x}}{(-x)}\right)=x\left(1-\frac{\sqrt{-x}}{(\sqrt{-x})^2}\right)=$$

$$=x\left(1-\frac{1}{\sqrt{-x}}\right)$$

Calcolare $\lim_{x\to 0}\frac{\tan x}{x}$

$$\lim_{x\to 0}\frac{\tan x}{x}=\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x}=\lim_{x\to 0}\underbrace{\frac{\sin x}{x}}_{1}\cdot\frac{1}{\underbrace{\cos x}_{1}}\overset{AL}{=}1$$

Calcolare $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

$$\boxed{\begin{gathered}\sin^2 x+\cos^2 x=1\\ \forall x\in\mathbb{R}\end{gathered}}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2}=$$

$$=\lim_{x\to 0}\frac{1-\cos x}{x^2}\cdot\frac{1+\cos x}{1+\cos x}=$$

$$=\lim_{x\to 0}\frac{1-\cos^2 x}{x^2(1+\cos x)}=\lim_{x\to 0}\frac{\sin^2 x}{x^2(1+\cos x)}=$$

$$=\lim_{x\to 0}\underbrace{\frac{\sin x}{x}}_{\to 1}\cdot\underbrace{\frac{\sin x}{x}}_{\to 1}\cdot\frac{1}{1+\underbrace{\cos x}_{\to 1}}=\frac{1}{2}$$

Definizione (infinitesimi dello stesso ordine)

Sia $x_0 \in \overline{\mathbb{R}}$ e siano f, g definite in un intorno I di x_0, escluso eventualmente x_0, con $g(x) \neq 0$ per ogni $x \in I \setminus \{x_0\}$.

Siano f e g infinitesime per $x \to x_0$.

(cioè $\lim\limits_{x \to x_0} f(x) = 0$ e $\lim\limits_{x \to x_0} g(x) = 0$)

Si dice che f e g sono infinitesime con lo stesso ordine per $x \to x_0$ se e solo se

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \ell \in \mathbb{R} \setminus \{0\}.$$

In questo caso, si scrive $f(x) \sim \ell \cdot g(x)$ per $x \to x_0$

<u>Esempi</u>

- $f(x) = 2x^2 + x^3$, $\quad g(x) = x^2$, $\qquad x_0 = 0$

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{2x^2 + x^3}{x^2} = \lim_{x \to 0} \frac{\cancel{x^2}(2 + x)}{\cancel{x^2}} = 2$$

$$\underbrace{f(x)}_{= 2x^2 + x^3} \sim 2 \cdot \underbrace{g(x)}_{= x^2} \quad \text{per } x \to 0$$

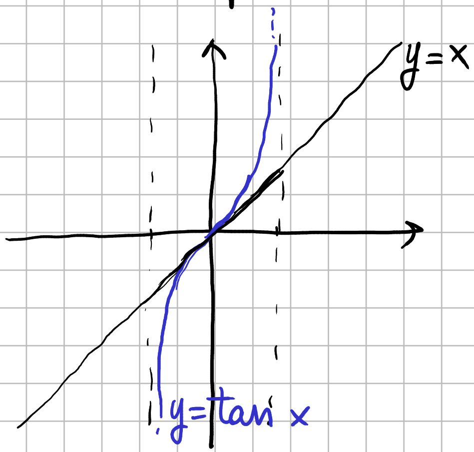
- $f(x) = 3x^4$, $\quad g(x) = 7x^3$, $\qquad x_0 = 0$

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{3\cancel{x^4}^{\,x}}{7\cancel{x^3}_{\,1}} = \lim_{x \to 0} \frac{3}{7} x = 0$$

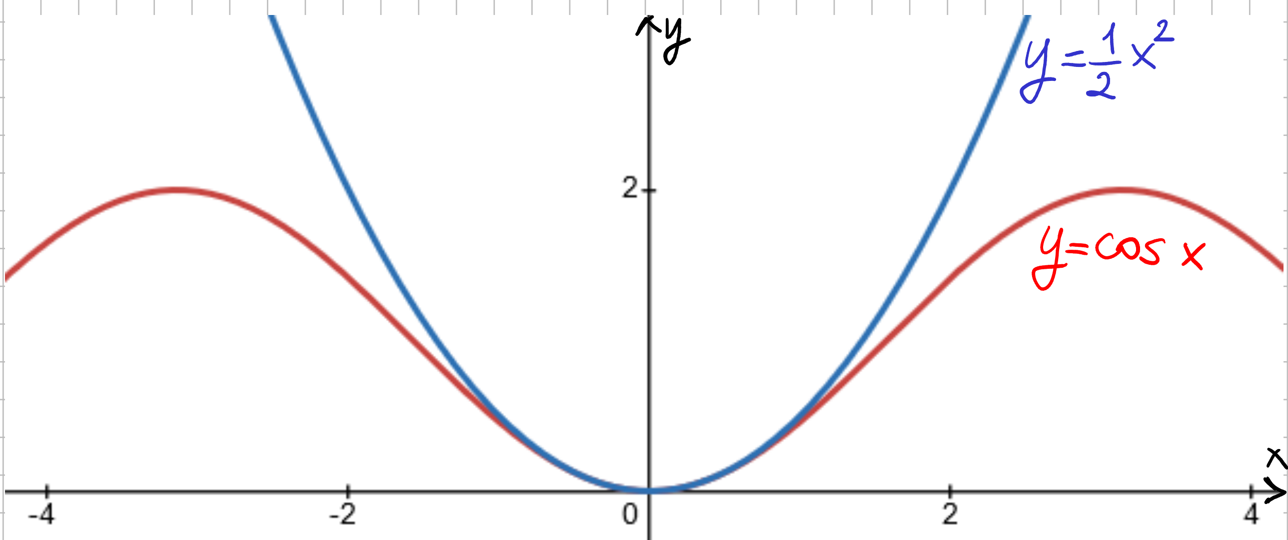
$3x^4$ e $7x^3$ NON sono infinitesimi dello stesso ordine per $x \to 0$

$\sin x \sim x$ per $x \to 0$ perchè $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\tan x \sim x$ per $x \to 0$ perchè $\lim_{x \to 0} \frac{\tan x}{x} = 1$



$(1 - \cos x) \sim \frac{1}{2} x^2$ per $x \to 0$ perchè $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$



Proposizione. Sia $x_0 \in \overline{\mathbb{R}}$ e siano f_1, f_2, g_1, g_2 funzioni definite in un intorno I di x_0, escluso eventualmente x_0, e infinitesime per $x \to x_0$.
Se $f_1 \sim f_2$ e $g_1 \sim g_2$ per $x \to x_0$, allora

$$f_1 \cdot g_1 \sim f_2 \cdot g_2 \quad \text{per } x \to x_0.$$

Se, in aggiunta, si ha $g_1(x) \neq 0 \ \forall x \in I \setminus \{x_0\}$ e $g_2(x) \neq 0 \ \forall x \in I \setminus \{x_0\}$, allora

$$\frac{f_1}{g_1} \sim \frac{f_2}{g_2} \quad \text{per } x \to x_0$$

Calcolare $\lim_{x \to 0} \frac{x \tan x}{1-\cos x}$

$\tan x \sim x$ per $x \to 0$ e $1-\cos x \sim \frac{1}{2}x^2$ per $x \to 0$

$$\lim_{x \to 0} \frac{x \cdot \tan x}{1-\cos x} = \lim_{x \to 0} \frac{\cancel{x} \cdot \cancel{x}}{\frac{1}{2}\cancel{x^2}} = 2$$

Calcolare $\lim_{x \to 0} \frac{\sin x - \tan x}{x^3}$

$$\lim_{x \to 0} \frac{\sin x - \tan x}{x^3} = \lim_{x \to 0} \frac{1}{x^3} \cdot \left(\sin x - \frac{\sin x}{\cos x} \right) =$$

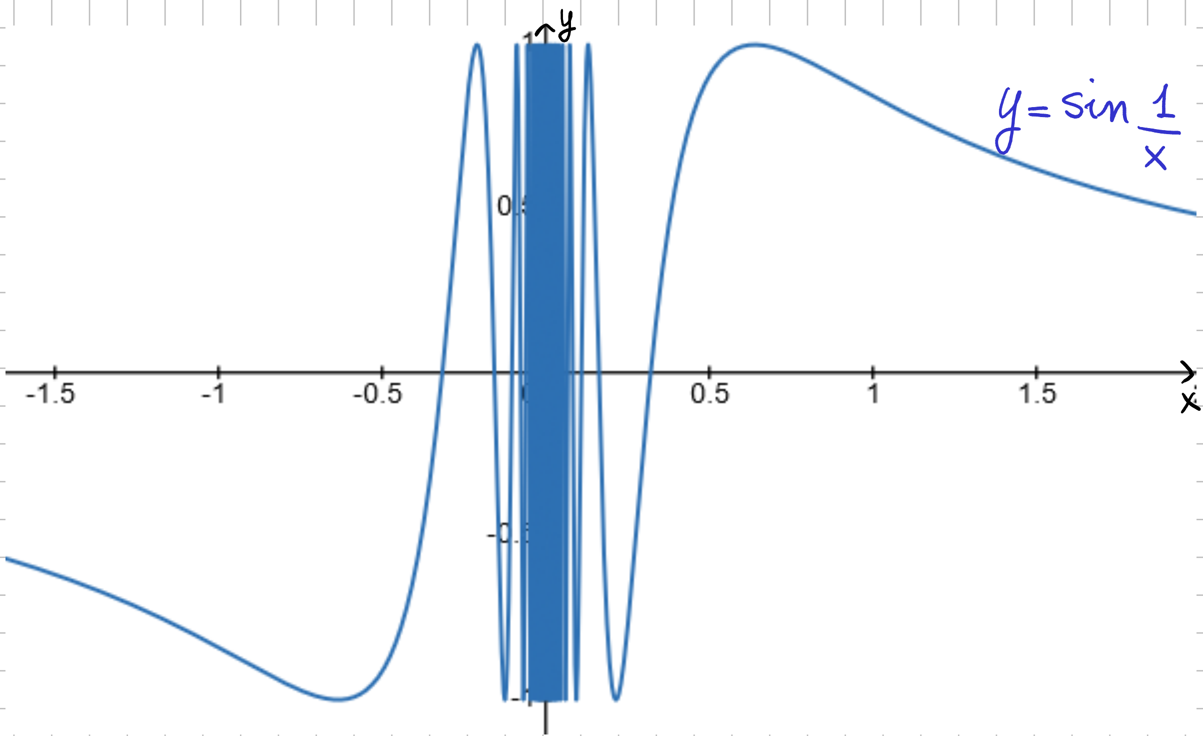
$$= \lim_{x \to 0} \frac{\sin x}{x^3} \cdot \left(1 - \frac{1}{\cos x} \right) =$$

$$= \lim_{x \to 0} \frac{\sin x}{x^3} \cdot \frac{\cos x - 1}{\cos x} =$$

$\sin x \sim x$ per $x \to 0$
$(\cos x - 1) \sim -\frac{1}{2}x^2$ per $x \to 0$

$$= \lim_{x\to 0} \frac{\cancel{x}}{\cancel{x^3}} \cdot \frac{-\frac{1}{2}\cancel{x^2}}{\cos x} = \lim_{x\to 0}\left(-\frac{1}{2}\cdot\frac{1}{\underset{\downarrow \atop 1}{\cos x}}\right) = -\frac{1}{2}$$

Calcolare $\lim_{x\to 0} x\cdot\sin\frac{2}{x}$



Per $x\to 0$ la funzione $y=\sin\frac{2}{x}$ non ammette limite

Osservazione. $f(x)=x\cdot\sin\frac{2}{x}$ è una funzione pari

Iniziamo a considerare $\lim_{x\to 0^+} x\cdot\sin\frac{2}{x}$

$$-1\le \sin\frac{2}{x}\le 1 \quad \forall x\in\mathbb{R}\setminus\{0\}$$

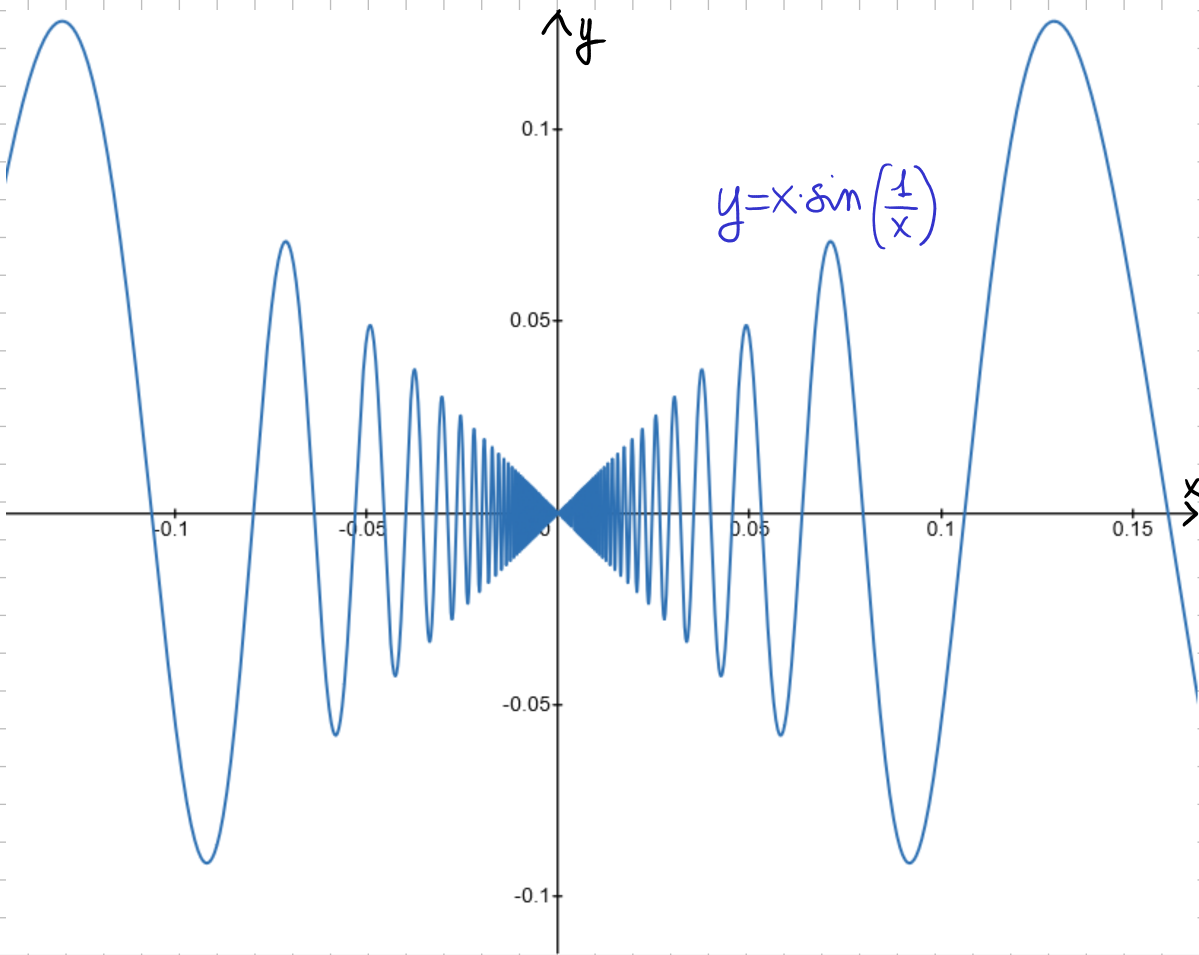
Moltiplichiamo tutti i membri per $x>0$

$$\underset{\downarrow \text{ per } x\to 0^+ \atop 0}{-x} \le x\cdot\sin\frac{2}{x} \le \underset{\downarrow \text{ per } x\to 0^+ \atop 0}{x} \quad \forall x\in(0,+\infty)$$

Dato che $\lim_{x\to 0^+}(-x) = \lim_{x\to 0^+} x = 0$, per il II teorema del confronto, si ha $\lim_{x\to 0^+} x \cdot \sin\frac{2}{x} = 0$.

Inoltre, poiché la funzione $f(x) = x \cdot \sin\frac{2}{x}$ è pari, si ha anche $\lim_{x\to 0^-} x \cdot \sin\frac{2}{x} = 0$.

In conclusione, $\lim_{x\to 0} x \cdot \sin\frac{2}{x} = 0$



Calcolare $\lim_{x\to+\infty} \frac{4x-9x^{3/4}}{5x+6\sin x}$

$$4x-9x^{3/4} = x\left(4-\frac{9}{x^{1/4}}\right) \sim 4x \quad \text{per } x\to+\infty$$

$$\frac{9}{x^{1/4}} \to 0 \quad \text{per } x\to+\infty$$

$$5x+6\sin x = x\left(5+6\frac{\sin x}{x}\right)$$

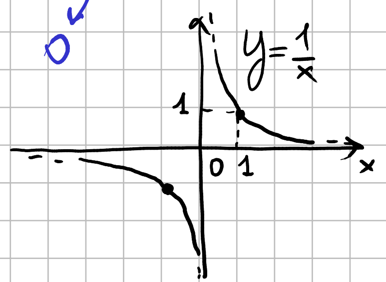
Calcoliamo $\lim_{x\to+\infty} \frac{\sin x}{x}$

$$-1 \leq \sin x \leq 1 \quad \forall x\in\mathbb{R}$$

Moltiplichiamo tutti i membri per $\frac{1}{x}$ con $x>0$

$$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \quad \forall x \in (0,+\infty)$$

$$-\frac{1}{x} \to 0 \qquad \frac{1}{x} \to 0$$



Dato che $\lim_{x\to+\infty}\left(-\frac{1}{x}\right) = \lim_{x\to+\infty}\frac{1}{x} = 0$, per il II teorema del confronto, si ha $\lim_{x\to+\infty}\frac{\sin x}{x} = 0$.

$$5x+6\sin x = x\left(5+6\frac{\sin x}{x}\right) \sim 5x \quad \text{per } x\to+\infty$$

$$\frac{\sin x}{x} \to 0 \quad \text{per } x\to+\infty$$

$$\lim_{x \to +\infty} \frac{4x - 9x^{3/4}}{5x + 6\sin x} = \lim_{x \to +\infty} \frac{4\cancel{x}}{5\cancel{x}} = \frac{4}{5}$$