Analisi Matematica 1 – Esercitazione del 25 settembre 2024

1. Determinare l'estremo superiore e l'estremo inferiore di ognuno dei seguenti insiemi. Stabilire anche se ciascun insieme ammette massimo e se ammette minimo.
   (a) $A = \{x \in \mathbb{R} : x^2 - 3 \leqslant 0\}$

   (b) $B = \{x^2 - 3 : x < 0\}$

   (c) $C = \{(1-x)^{\sqrt{3}} : x \in (0,1]\}$

   (d) $D = \{|2^{|x|} - 2| : 0 \leqslant x \leqslant 2\}$

   (e) $E = \{x \in \mathbb{R} : \log|x+3| > 0\} \cap \{x \in \mathbb{R} : x^5 + 2x^4 \leqslant 0\}$

2. Determinare il dominio della funzione $f\colon \operatorname{dom}(f) \subseteq \mathbb{R} \to \mathbb{R}$ definita da
$$f(x) = \log(9x - 2x^2 - 4) + \frac{1}{\sqrt{e^{2x+1} - e^x}} - \sqrt{\frac{(4x^2 - 20x + 25)(4x^2 - x - 3)}{[(x^3+1)^4 + \sqrt{7}](3x - 1 - 4x^2)}}.$$

3. Si consideri la funzione $f\colon \operatorname{dom}(f) \subseteq \mathbb{R} \to \mathbb{R}$ definita da
$$f(x) = \left(1 - e^{\sqrt{2x^2+x}}\right)(1 + \log|x|).$$
   Determinare il dominio di f e l'insieme $A = \{x \in \operatorname{dom}(f) : f(x) \leqslant 0\}$.

4. (Tema d'esame 4 settembre 2024) Si consideri la funzione $f\colon \operatorname{dom}(f) \subseteq \mathbb{R} \to \mathbb{R}$ definita da
$$f(x) = \frac{6\sqrt{2\pi - x}}{\cos x} \log\left(\frac{x}{|x|+1}\right).$$
   Determinare il dominio di f e per quali $x \in \operatorname{dom}(f)$ si ha $f(x) > 0$.

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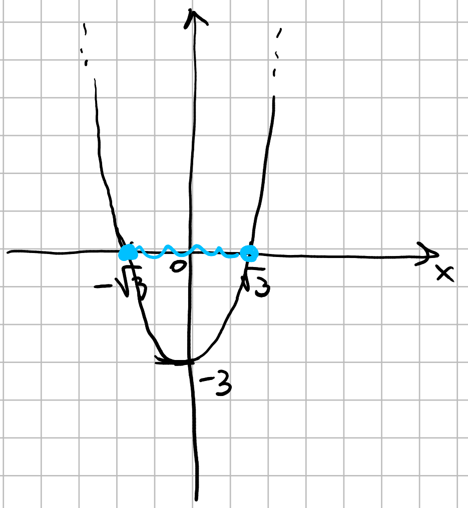
$A = \{x \in \mathbb{R} : x^2 - 3 \leq 0\} = [-\sqrt{3}, \sqrt{3}]$

$f(x) = x^2 - 3$

$x^2 - 3 = 0 \iff x^2 = 3 \iff x = \pm\sqrt{3}$



Ogni numero reale $\leq -\sqrt{3}$ è un minorante di A

$\inf A = -\sqrt{3}$

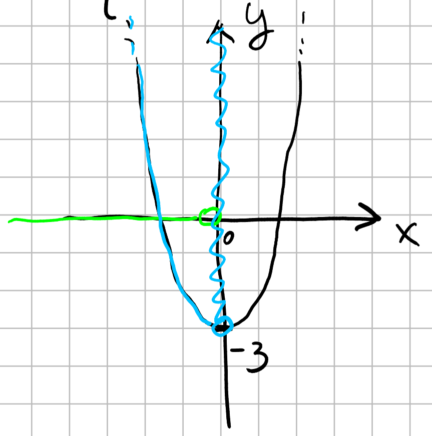
Dato che $-\sqrt{3} \in A$, si ha $\min A = -\sqrt{3}$

Ogni numero reale $\geq \sqrt{3}$ è un maggiorante di A

$\sup A = \sqrt{3}$

$\sqrt{3} \in A$, quindi $\max A = \sqrt{3}$

$B = \{x^2 - 3 : x < 0\} = (-3, +\infty)$



Nessun numero reale è un maggiorante di B.

Quindi $\sup B = +\infty$ e B non ammette massimo.

Ogni numero reale ≤ -3 è minorante di B.

$\inf B = -3$

$-3 \notin B$, quindi B non ammette minimo.

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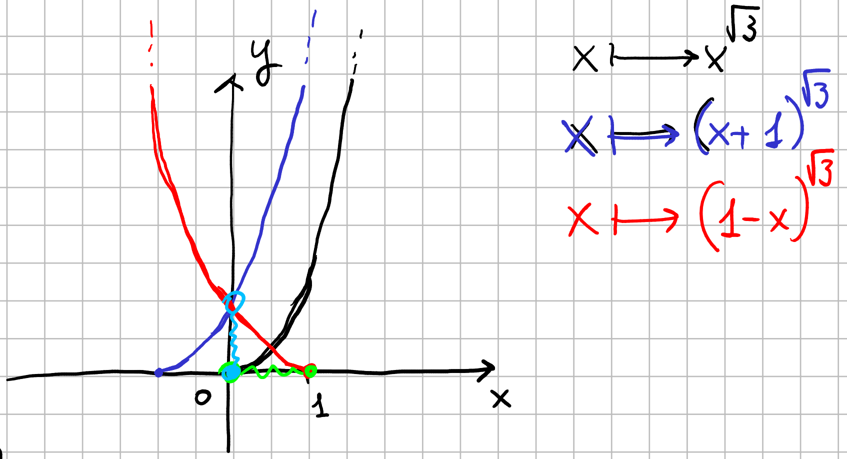
$$C = \left\{ (1-x)^{\sqrt{3}} : x \in (0,1] \right\} = [0,1)$$

$$f(x) = (1-x)^{\sqrt{3}}$$



Ogni numero reale ≤ 0 è un minorante di C

$\inf C = 0$

$0 \in C$, quindi $\min C = 0$

Ogni numero reale ≥ 1 è un maggiorante di C

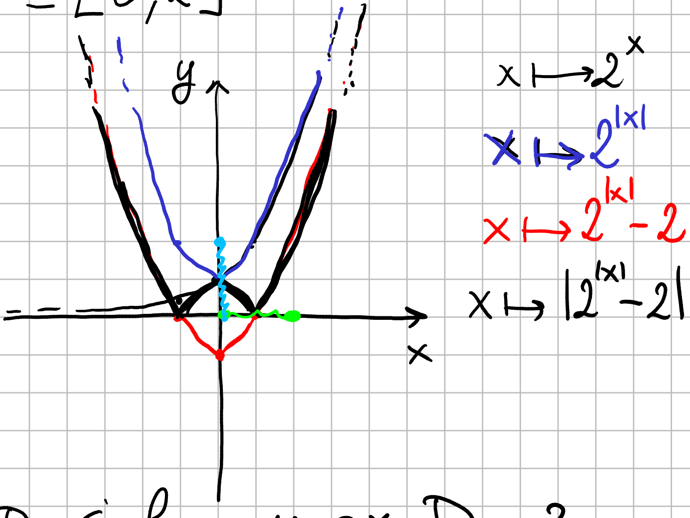
$\sup C = 1$

$1 \notin C$, quindi C non ammette massimo

* * *

$$D = \left\{ \left|2^{|x|} - 2\right| : 0 \leq x \leq 2 \right\} = [0,2]$$

$$f(x) = \left|2^{|x|} - 2\right|$$



$\inf D = 0$

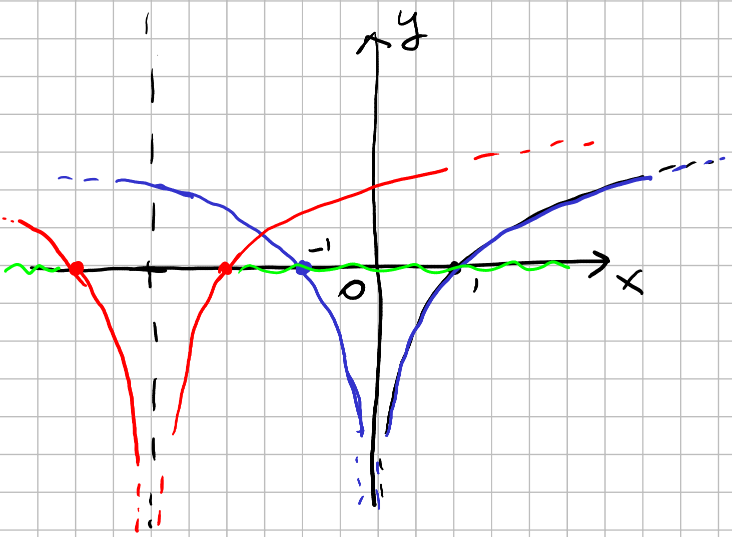
$0 \in D$, quindi $\min D = 0$

$\sup D = 2$. Dato che $2 \in D$, si ha $\max D = 2$

$$E = \underbrace{\{x \in \mathbb{R} : \log|x+3| > 0\}}_{E_1} \cap \underbrace{\{x \in \mathbb{R} : x^5 + 2x^4 \le 0\}}_{E_2}$$

$$E_1 = \{x \in \mathbb{R} : \log|x+3| > 0\}$$

$\boxed{\log \text{ INDICA IL LOGARITMO NATURALE}}$



$x \longmapsto \log x$

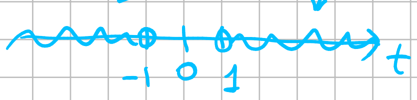
$x \longmapsto \log|x|$

$x \longmapsto \log|x+3|$

$$\log|x+3| > 0$$

$$|x+3| > 1$$

$$|t| > 1 \iff t < -1 \lor t > 1$$



$$x+3 < -1 \lor x+3 > 1$$

$$x < -4 \lor x > -2$$

$$E_1 = (-\infty, -4) \cup (-2, +\infty)$$

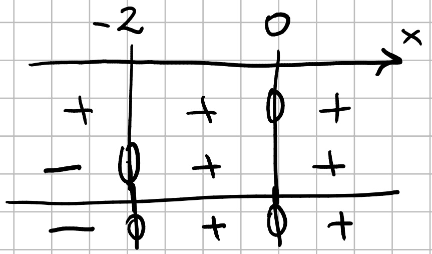
$$E_2 = \{x \in \mathbb{R} : x^5 + 2x^4 \le 0\} = (-\infty, -2] \cup \{0\}$$

$$x^5 + 2x^4 \le 0$$

$$x^4(x+2) \le 0$$

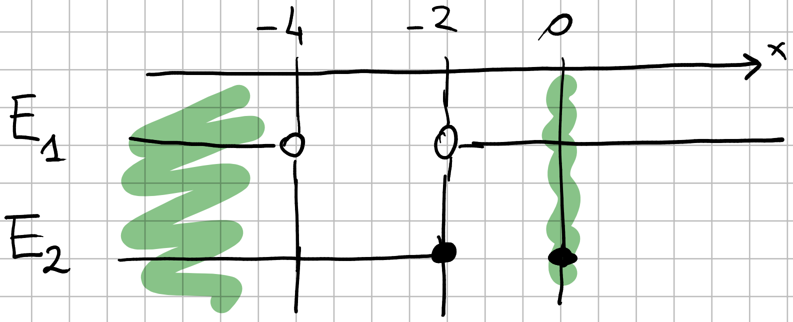
$x^4 \ge 0 \ \forall x \in \mathbb{R}$ $\quad (x^4 = 0 \iff x = 0)$

$x+2 \ge 0 \iff x \ge -2$



$$x \le -2 \lor x = 0$$

$$E = \left((-\infty, -4) \cup (-2, +\infty) \right) \cap \left((-\infty, -2] \cup \{0\} \right)$$



$$E = (-\infty, -4) \cup \{0\}$$

Nessun numero reale è un minorante di E

inf $E = -\infty$

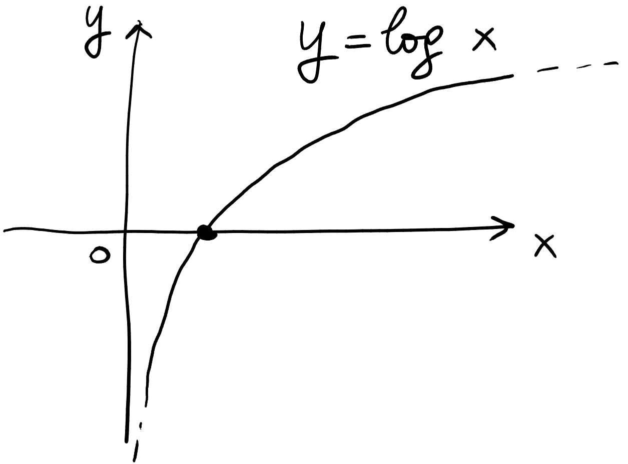
E non ammette minimo

Ogni numero reale ≥ 0 è un maggiorante di E

sup $E = 0$

$0 \in E$, quindi max $E = 0$

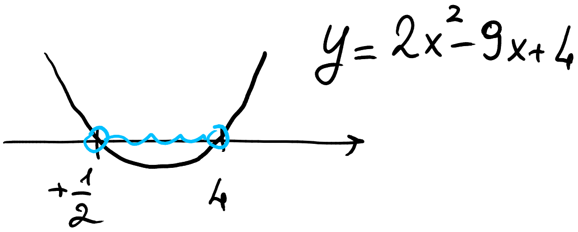
Determinare il dominio della funzione f definita da $f(x) = \log(9x-2x^2-4)$



$$9x-2x^2-4>0 \iff 2x^2-9x+4<0 \iff \frac{1}{2}<x<4$$

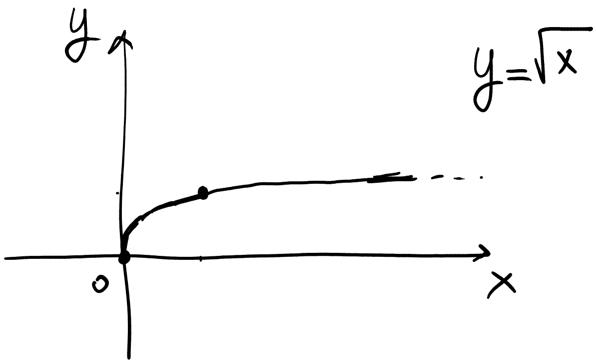
$$2x^2-9x+4=0 \iff x=\frac{9\pm7}{4} \iff x=4 \vee x=\frac{1}{2}$$

$$\Delta=(-9)^2-4\cdot2\cdot4=7^2$$



$$\mathrm{dom}(f)=\left(\frac{1}{2},4\right)$$

Determinare il dominio della funzione f definita da $f(x)=\sqrt{\dfrac{(4x^2-20x+25)(4x^2-x-3)}{[(x^3+1)^4+\sqrt{7}](3x-1-4x^2)}}$



$$\frac{(4x^2-20x+25)(4x^2-x-3)}{[(x^3+1)^4+\sqrt{7}](3x-1-4x^2)} \geq 0$$

$$4x^2-20x+25 \geq 0$$

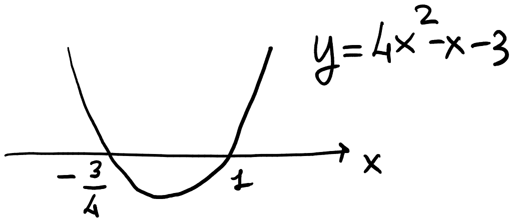
$$(2x-5)^2 \geq 0 \quad \forall x \in \mathbb{R}$$

$(2x-5)^2$ si annulla per $x = \frac{5}{2}$

$$4x^2-x-3 \geq 0 \iff x \leq -\frac{3}{4} \vee x \geq 1$$

$$\Delta = 1+48 = 7^2$$

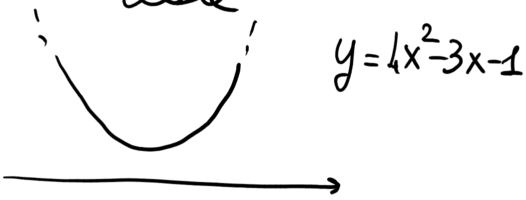
$$4x^2-x-3=0 \iff x = \frac{1 \pm 7}{8} \iff x = 1 \vee x = -\frac{3}{4}$$



$$\underbrace{(x^3+1)^4}_{\geq 0 \ \forall x \in \mathbb{R}} + \overset{>0}{\boxed{\sqrt{7}}} > 0 \quad \forall x \in \mathbb{R}$$

$$3x-1-4x^2 > 0 \iff 4x^2-3x+1 < 0$$ nessuna soluzione reale

$$\Delta = (-3)^2 - 4 \cdot 4 \cdot 1 = 9-16 < 0$$



		$-\frac{3}{4}$		1		$\frac{5}{2}$	
$(2x-5)^2$	+		+		+	0	+
$4x^2-x-3$	+	0	−	0	+		+
Den.	−		−		−		−
	−	0	+	0	−	0	−

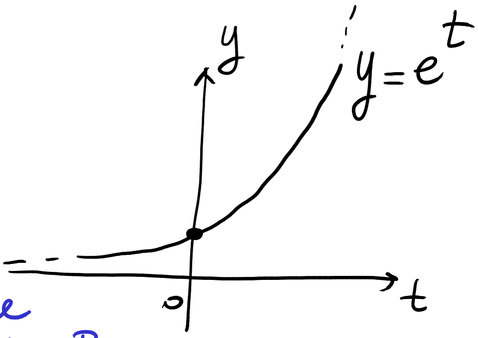
$$-\frac{3}{4} \leq x \leq 1 \quad \vee \quad x = \frac{5}{2}$$

$$\text{dom}(f) = \left[-\frac{3}{4}, 1\right] \cup \left\{\frac{5}{2}\right\}$$

Determinare il dominio della funzione f definita da

$$f(x) = \log(9x - 2x^2 - 4) + \frac{1}{\sqrt{e^{2x+1} - e^x}} - \sqrt{\frac{(2x-5)^2(4x^2-x-3)}{[(x^3+1)^4+\sqrt{7}](3x-1-4x^2)}}$$

$$\begin{cases} 9x - 2x^2 - 4 > 0 \\ e^{2x+1} - e^x > 0 \\ \dfrac{(2x-5)^2(4x^2-x-3)}{[(x^3+1)^4+\sqrt{7}](3x-1-4x^2)} \geq 0 \end{cases} \iff \begin{cases} \frac{1}{2} < x < 4 \\ x > -1 \\ -\frac{3}{4} \leq x \leq 1 \lor x = \frac{5}{2} \end{cases}$$



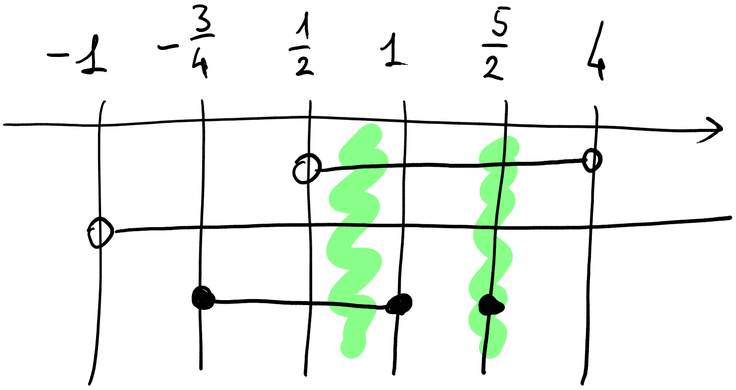
$$e^{2x+1} - e^x > 0$$

$$e^{2x+1} > e^x$$

La funzione $y = e^t$, con $t \in \mathbb{R}$, è strett. crescente

$$2x + 1 > x$$

$$x > -1$$

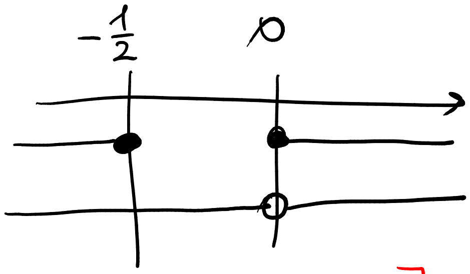


$$\mathrm{dom}(f) = \left(\frac{1}{2}, 1\right] \cup \left\{\frac{5}{2}\right\}$$

Determinare il dominio della funzione f definita da $f(x) = \left(1 - e^{\sqrt{2x^2+x}}\right)\left(1 + \log|x|\right)$ e determinare l'insieme $A = \{x \in \text{dom}(f) : f(x) \leq 0\}$.

$$\begin{cases} 2x^2 + x \geq 0 \\ |x| > 0 \end{cases} \iff \begin{cases} x(2x+1) \geq 0 \\ x \neq 0 \end{cases} \iff \begin{cases} x \leq -\frac{1}{2} \vee x \geq 0 \\ x \neq 0 \end{cases}$$

$$\boxed{|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}}$$



$$\text{dom}(f) = \left(-\infty, -\frac{1}{2}\right] \cup (0, +\infty)$$

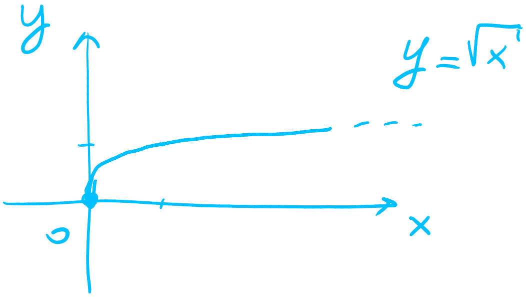
$$\left(1 - e^{\sqrt{2x^2+x}}\right)\left(1 + \log|x|\right) \leq 0$$

$$1 - e^{\sqrt{2x^2+x}} \geq 0$$

$$e^{\sqrt{2x^2+x}} \leq 1$$

$$e^{\sqrt{2x^2+x}} \leq e^0$$

$$\sqrt{2x^2+x} \leq 0$$



$$\sqrt{2x^2+x} = 0$$

$$2x^2 + x = 0$$

$$x(2x+1) = 0$$

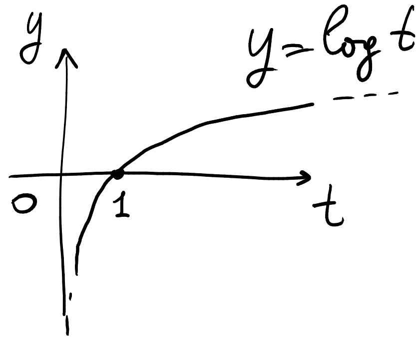
$$x = 0 \vee x = -\frac{1}{2}$$

$$1+\log|x|\geq 0$$

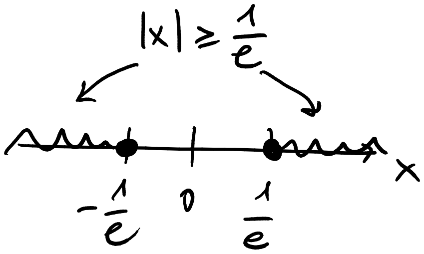
$$\log|x|\geq -1$$

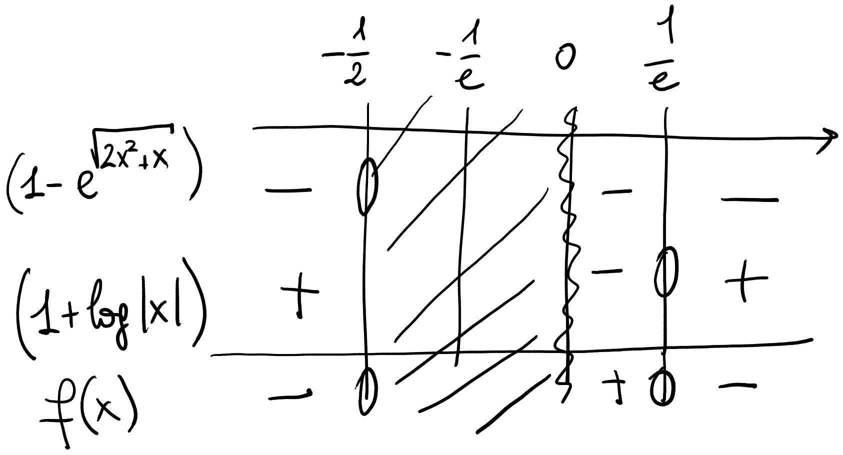
$$\log|x|\geq \log e^{-1}$$

$$|x|\geq \frac{1}{e}$$



$$x\leq -\frac{1}{e} \vee x\geq \frac{1}{e}$$





$$f(x)\leq 0 \Leftrightarrow x\leq -\frac{1}{2} \vee x\geq \frac{1}{e}$$

$$A=\left(-\infty,-\frac{1}{2}\right]\cup\left[\frac{1}{e},+\infty\right)$$

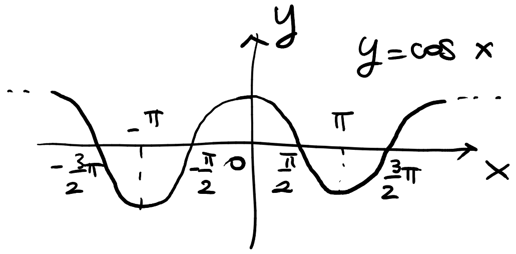
(4 settembre 2024)

$$f:\operatorname{dom}(f)\subseteq\mathbb{R}\longrightarrow\mathbb{R}$$

$$f(x)=\frac{6\sqrt{2\pi-x}}{\cos x}\cdot\log\left(\frac{x}{|x|+1}\right)$$

- Determinare il dominio di f
- Determinare per quali $x\in\operatorname{dom}(f)$ si ha $f(x)>0$

$$\begin{cases} 2\pi-x\geq 0 \\ \cos x\neq 0 \\ \dfrac{x}{|x|+1}>0 \\ |x|+1\neq 0 \end{cases}$$



- $2\pi - x \geq 0 \iff x \leq 2\pi$

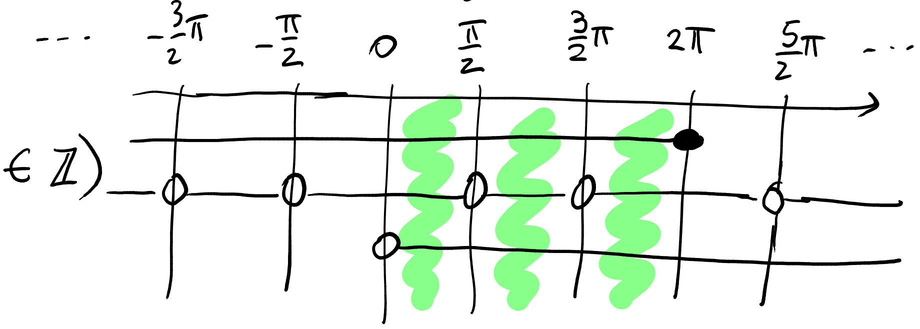
- $\cos x \neq 0 \iff x \neq \frac{\pi}{2} + k\pi \quad (k \in \mathbb{Z})$

- $|x| + 1 \neq 0 \iff |x| \neq -1 \quad \forall x \in \mathbb{R}$

$\underset{\geq 0 \ \forall x \in \mathbb{R}}{\boxed{|x|}} + \overset{>0}{\boxed{1}} > 0 \quad \forall x \in \mathbb{R}$

- $\frac{x}{|x|+1} > 0 \iff x > 0$ perchè il denominatore è maggiore di 0 $\forall x \in \mathbb{R}$

$$\begin{cases} x \leq 2\pi \\ x \neq \frac{\pi}{2} + k\pi \quad (k \in \mathbb{Z}) \\ x > 0 \end{cases}$$



$\mathrm{dom}(f) = \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$

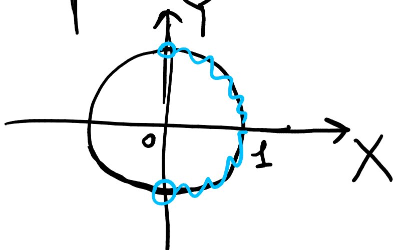
- Positività di f

- $\sqrt{2\pi - x} > 0 \iff x < 2\pi$

La radice quadrata è >0 per ogni x nel suo dominio, esclusi i valori di x per cui il radicando si annulla

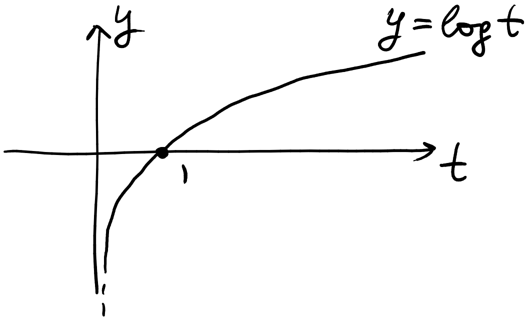
- $\cos x > 0$

Oss. Il dominio di f è contenuto in $[0, 2\pi]$, quindi si può risolvere $\cos x > 0$ per $x \in [0, 2\pi]$



Se $x \in [0, 2\pi]$, allora $\cos x > 0$ per $0 \leq x < \frac{\pi}{2} \vee \frac{3\pi}{2} < x \leq 2\pi$

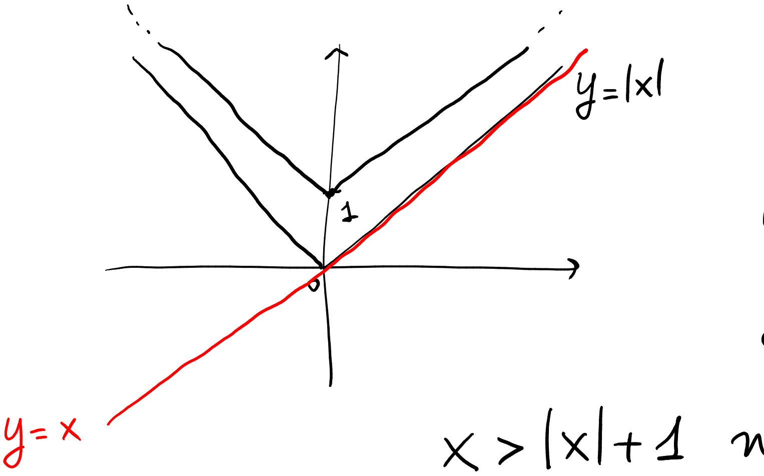
- $\log\left(\frac{x}{|x|+1}\right) > 0$



$$\frac{x}{|x|+1} > 1$$

$|x|+1 > 0 \ \forall x \in \mathbb{R}$

$$x > |x| + 1$$



Qualunque sia $x \in \mathbb{R}$ il grafico di $y = x$ è "al di sotto" del grafico di $y = |x| + 1$

$x > |x| + 1$ non ha soluzioni reali

	0		$\frac{\pi}{2}$		$\frac{3\pi}{2}$		2π
$\sqrt{2\pi - x}$		+		+		+	0
$\cos x$		+		−		+	
$\log\left(\frac{x}{\lvert x\rvert+1}\right)$		−		−		−	
$f(x)$		−		+		−	0

$f(x) > 0 \iff x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$