• $\displaystyle\sum_{n=2}^{\infty} \frac{n^2+\sqrt{\log(n+2)}}{n^3\cdot\sqrt{\log(n+2)}}$ $\qquad a_n = \dfrac{n^2+\sqrt{\log(n+2)}}{n^3\cdot\sqrt{\log(n+2)}} \geq 0$ serie a t.p.

• $n^2+\sqrt{\log(n+2)} \sim n^2$ per $n \to \infty$

• $n^3\cdot\sqrt{\log(n+2)} \sim n^3\sqrt{\log n}$ per $n \to \infty$

$$a_n \sim \frac{n^2}{n^3\sqrt{\log n}} = \frac{1}{n(\log n)^{1/2}} = b_n$$

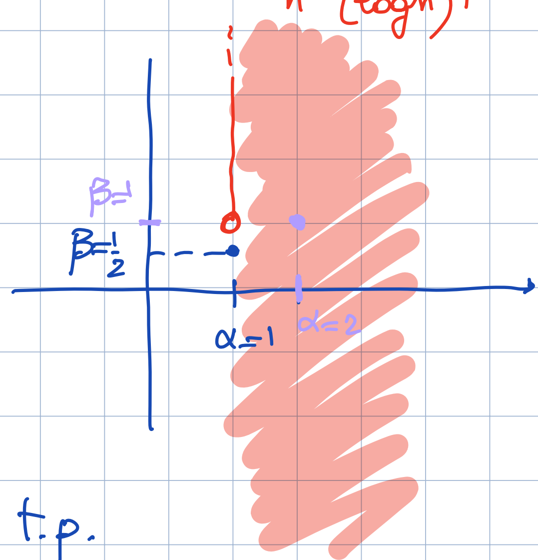
? $\displaystyle\sum_{n=2}^{\infty} b_n = \sum_{n=2}^{\infty} \frac{1}{n(\log n)^{1/2}}$ diverge

$\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{\alpha}(\log n)^{\beta}}$

$\alpha = 1$

$\beta = \frac{1}{2}$

per il criterio del confronto asintotico diverge anche la serie data



• $\displaystyle\sum_{n=2}^{\infty} \underbrace{\frac{n^2+3}{n^4\cdot\log n}}_{a_n}$ $\qquad a_n \geq 0$ t.p.

• $n^2+3 \sim n^2$ per $n \to \infty$

• $n^4\cdot\log n$

$$a_n \sim \frac{n^2}{n^4\log n} = \frac{1}{n^2\log n} = b_n$$

$\displaystyle\sum_{n=2}^{\infty} b_n = \sum_{n=2}^{\infty} \frac{1}{n^2\log n}$ converge

$\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{\alpha}(\log n)^{\beta}}$

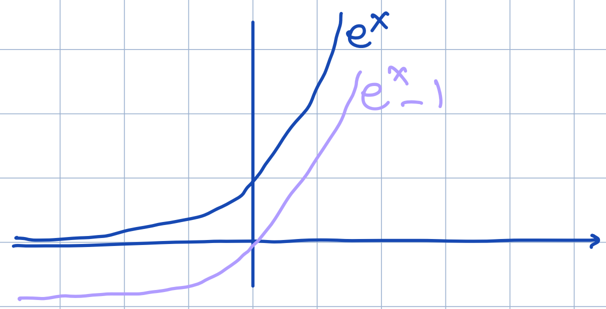
$\alpha = 2$

$\beta = 1$

$\Rightarrow$ per il criterio del conf as, converge anche la serie data

$$\sum_{n=1}^{\infty} \underbrace{\left(\frac{n}{n+1}\right)^{n^2}\left[\frac{n}{2}\left(e^{1/n}-1\right)\right]^{n}}_{a_n}$$

$e^x - 1 > 0$ quando $x > 0$



$\Rightarrow a_n \geqslant 0 \quad \forall n \geqslant 1$ t.p.

$e^{1/n} - 1 > 0 \quad \forall n \geqslant 1$

$$a_n = \left(\left(\frac{n}{n+1}\right)^{n}\cancel{[}\frac{n}{2}\left(e^{1/n}-1\right)\cancel{]}\right)^{n}$$

applico il criterio della radice e calcolo

$$\ell = \lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \sqrt[\cancel{n}]{\left(\left(\frac{n}{n+1}\right)^{n}\cdot\frac{n}{2}\left(e^{1/n}-1\right)\right)^{\cancel{n}}} =$$

$$= \lim_{n\to\infty} \underbrace{\left(\frac{n}{n+1}\right)^{n}}_{\to \frac{1}{e}} \frac{n}{2}\left(e^{1/n}-1\right) \overset{(*)}{=}$$

- $\left(\frac{n}{n+1}\right)^{n} = \left(\frac{1}{\frac{n+1}{n}}\right)^{n} = \frac{1}{\left(1+\frac{1}{n}\right)^{n}} \xrightarrow[n\to\infty]{} \frac{1}{e}$

- $e^{1/n} - 1 \sim \frac{1}{n}$ per $n \to \infty$

$\overset{(*)}{=} \lim_{n\to\infty} \frac{1}{e}\,\frac{\cancel{n}}{2}\cdot\frac{1}{\cancel{n}} = \frac{1}{2e} < 1 \Rightarrow$ per il criterio della radice la serie data converge

- $\displaystyle\sum_{n=1}^{\infty} \underbrace{\frac{\alpha^n + \arctan(n!)}{e^{2n} - 1}}_{a_n}$, $\alpha \in (0, +\infty)$

$\alpha^n \geq 0$, $\arctan(n!) \geq 0$
numeratore ≥ 0

$e^{2n} - 1 > 0 \quad \forall n > 0$

$\Rightarrow a_n \geq 0$ la serie è a.t.p.

- $\alpha^n + \arctan(n!)$

$$\lim_{n\to\infty} \alpha^n = \begin{cases} 0 & \text{se } 0<\alpha<1 \\ 1 & \text{se } \alpha = 1 \\ +\infty & \text{se } \alpha > 1 \end{cases}$$

$$\alpha^n + \arctan(n!) \sim \begin{cases} \frac{\pi}{2} & \text{se } 0<\alpha<1 \\ 1 + \pi/2 & \text{se } \alpha = 1 \\ \alpha^n & \text{se } \alpha > 1 \end{cases}$$

- $e^{2n} - 1 \sim e^{2n}$ per $n \to \infty$

$$a_n \sim \underbrace{\begin{cases} \frac{\pi/2}{e^{2n}} & \text{se } 0<\alpha<1 \\ \frac{1+\pi/2}{e^{2n}} & \text{se } \alpha = 1 \\ \frac{\alpha^n}{e^{2n}} & \text{se } \alpha > 1 \end{cases}}_{b_n}$$

$a_n \sim b_n$ per $n \to \infty$

1) $\sum b_n$ $\quad \boxed{0<\alpha<1}$ $\quad \sum b_n = \sum \frac{\pi/2}{e^{2n}} = \frac{\pi}{2} \sum \left(\frac{1}{e^2}\right)^n$

serie geometrica $\sum q^n$ con $q = \frac{1}{e^2} \in (-1,1) \Rightarrow$ serie geom convergente

per il criterio del conf asintotico converge anche la serie data perché $b_n \sim a_n$ per $n\to\infty$ e quindi $\sum a_n \sim \sum b_n$

$\alpha = 1$

2) $\sum b_n = \sum \frac{1+\pi/2}{e^{2n}} = \left(1+\frac{\pi}{2}\right)\cdot \sum \frac{1}{e^{2n}} = \left(1+\frac{\pi}{2}\right)\cdot \sum \left(\frac{1}{e^2}\right)^n$

ancora serie geometrica conv, per il criterio del confronto as. converge anche la serie data

3) $\underline{\alpha > 1}$, $\sum b_n = \sum \frac{\alpha^n}{e^{2n}} = \sum \left(\frac{\alpha}{e^2}\right)^n$

serie geometrica con $q = \frac{\alpha}{e^2}$, converge se $0 < \frac{\alpha}{e^2} < 1$

so che $\alpha > 0$

cioè se $0 < \alpha < e^2$, che a sistema con $\alpha > 1$ dà $1 < \alpha < e^2$

diverge se $\alpha \geq e^2$.

per il criterio del confronto asintotico ho

$\sum a_n$ converge se $1 < \alpha < e^2$

diverge se $\alpha \geq e^2$

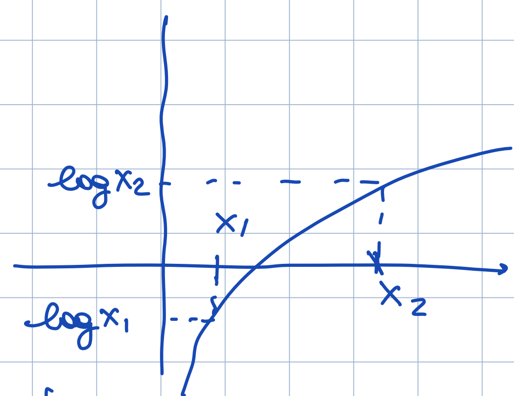
In conclusione $\sum a_n$ converge per $0 < \alpha < e^2$

diverge per $\alpha \geq e^2$

• $\underbrace{\sum_{n=1}^{\infty} n^n \cdot \left[\log\left(\overbrace{(2n)!+7}^{x_2}\right) - \log\left(\overbrace{(2n)!}^{x_1}\right)\right]}_{a_n}$

se $x_1 < x_2$

$\Rightarrow \log(x_1) < \log(x_2)$



$\Rightarrow a_n \geq 0 \quad \forall n \geq 1$ serie a.t.p.

- $\log((2n)!+7) - \log((2n)!) =$ $\qquad \log a - \log b = \log \frac{a}{b}$

$$= \log\left(\frac{(2n)! + 7}{(2n)!}\right) = \log\left(1 + \underbrace{\frac{7}{(2n)!}}_{\to 0 \text{ per } n\to\infty}\right) \sim \frac{7}{(2n)!}$$

$\log(1+x) \sim x$ se $x \to 0$

$$a_n \sim n^n \cdot \frac{7}{(2n)!} = b_n$$

studio $\sum b_n = \sum \frac{7n^n}{(2n)!} = 7 \sum_{n=1}^{\infty} \underbrace{\frac{n^n}{(2n)!}}_{c_n}$

studio $\sum c_n$ con il criterio del rapporto , $c_n > 0$

calcolo $\ell = \lim_{n\to\infty} \frac{c_{n+1}}{c_n} = \lim_{n\to\infty} \frac{\frac{(n+1)^{n+1}}{(2n+2)!}}{\frac{n^n}{(2n)!}} = \lim_{n\to\infty} \frac{(n+1)^{n+1}}{(2n+2)!} \cdot \frac{(2n)!}{n^n} =$

$$= \lim_{n\to\infty} \frac{(n+1)^n \overbrace{(n+1)}^{\sim n} \cancel{(2n)!}}{\underbrace{(2n+2)(2n+1)}_{\sim 4n^2}\cancel{(2n)!}\, n^n} = \lim_{n\to\infty} \frac{e \cdot n}{4n^2} = 0 < 1$$

$\Rightarrow \sum c_n$ converge per il criterio del rapporto

$\Rightarrow \sum b_n = 7 \sum c_n$ converge

$\Rightarrow \sum a_n$ conv per il criterio del confronto asintotico.