- $$\lim_{n\to\infty} \frac{\log\left[\left(3e^{n}\right)^{n}\right] - n^2}{\log\left(\frac{3+e^{2n}}{n}\right) - n} = \frac{\infty - \infty}{}$$ forma in det.

$\log\left[\left(3e^{n}\right)^{n}\right] \to \infty$, $\left(3e^{n}\right)^{n} \to \infty$, $e^{n} \to \infty$, $n^2 \to \infty$, $\frac{3+e^{2n}}{n} \to \frac{\infty}{\infty}$

$$\log(a\cdot b) = \log a + \log b$$

$$N = \log\left[\left(3e^{n}\right)^{n}\right] - n^2 = n \underbrace{\log\left(3e^{n}\right)}_{\log 3 + \log e^{n}} - n^2 =$$

$$= n\left(\log 3 + n\underset{=1}{(\log e)}\right) - n^2 =$$

$$= n\log 3 + \cancel{n^2} - \cancel{n^2} = n \cdot \log 3$$

$$D = \log\left(\frac{3+e^{2n}}{n}\right) - n = \log\left(3+e^{2n}\right) - \log n - n$$

$$\left(\log\left(\frac{a}{b}\right) = \log a - \log b\right)$$

$$\sim \log\left(e^{2n}\right) - \log n - n =$$

per confronto tra 3 e e^{2n}

$$= 2n \underset{=1}{(\log e)} - \log n - n$$

$$= n - \log n \sim n$$

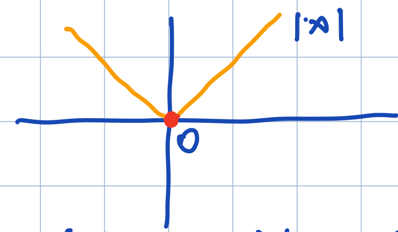
$$\log\left(\frac{3+e^{2n}}{n}\right) \sim n \quad \text{per } n \to \infty$$

$$\lim_{n\to\infty} \overset{\text{dato}}{\cdots\cdots} = \lim_{n\to\infty} \frac{\cancel{n}\cdot\log 3}{\cancel{n}} = \log 3$$

• $f(x) = \sqrt{|e^{x-2}-3|} - 2x$

• dom (f) : $|e^{x-2}-3| \geq 0$ (radice quad) $\forall x \in \mathbb{R}$

dom $(f) = \mathbb{R} = (-\infty, +\infty)$



$|x| \geq 0 \quad \forall x \in \mathbb{R}$

$|x| > 0 \quad x \neq 0$

• sim: $f(-x) = \sqrt{|e^{-x-2}-3|} + 2x$

$\neq f(x)$

$\neq -f(x)$

$\nexists$ simmetrie.

• $\lim\limits_{x \to -\infty} f(x) = \lim\limits_{x \to -\infty} \sqrt{|e^{\overset{-\infty}{x-2}}-3|} - \underset{-\infty}{2x} = \sqrt{3} - (-\infty) = +\infty$

$\quad \to 0$

cerco l'as. obl

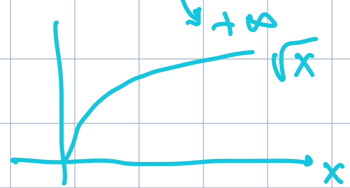
$m_- = \lim\limits_{x \to -\infty} \frac{f(x)}{x} = \lim\limits_{x \to -\infty} \frac{\overset{\sqrt{3}}{\sqrt{|e^{x-2}-3|}} - 2x}{x} = \lim\limits_{x \to -\infty} \underset{\to 0}{\frac{\sqrt{3}}{x}} - 2 =$

$m_- = -2$

$q_- = \lim\limits_{x \to -\infty} (f(x) - m_- x) = \lim\limits_{x \to -\infty} \overset{\sqrt{3}}{\sqrt{|e^{x-2}-3|}} \cancel{-2x} \cancel{+2x} = \sqrt{3}$

$\Rightarrow \quad y = -2x + \sqrt{3}$ è as. obl sx

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to +\infty} \underset{\to +\infty}{\sqrt{|e^{\overset{+\infty}{x-2}}-3|}} - \underset{+\infty}{2x} = +\infty - \infty$



$$= \lim_{x\to+\infty}\left(\frac{e^{x/2}}{e}-2x\right)=\lim_{x\to+\infty}\frac{e^{x/2}}{e}\left(1-\underbrace{\frac{2x\cdot e}{e^{x/2}}}_{\to 0}\right)=\lim_{x\to+\infty}\frac{e^{x/2}}{e}=+\infty$$

$$\sqrt{|e^{x-2}-3|}\underset{\text{per } x\to+\infty}{\sim}\sqrt{|e^{x-2}|}=\sqrt{e^{x-2}}=e^{\frac{x}{2}-1}=\frac{e^{x/2}}{e}$$

l'exp(x) → ∞ + velocemente di una qualsiasi potenza di x quando $x\to\infty$

Calcolo $\lim_{x\to+\infty}\frac{2xe}{e^{x/2}}=\frac{\infty}{\infty}$

$$\underset{(H)}{=}\lim_{x\to+\infty}\frac{2e}{e^{x/2}\cdot\frac{1}{2}}=0$$

$$\lim_{x\to+\infty}\frac{e^{\beta x}}{x^{\alpha}}=+\infty$$

$\alpha,\beta>0$

? as obliquo dx

$$m_+=\lim_{x\to+\infty}\frac{\sqrt{|e^{x-2}-3|}-2x}{x}=\lim_{x\to+\infty}\frac{\sqrt{|e^{x-2}-3|}}{x}-2=+\infty$$

$\sqrt{|e^{x-2}-3|}\sim\frac{e^{x/2}}{e}$

per confronto di ∞

∄ as obl destro

$$f(x)=\sqrt{|e^{x-2}-3|}-2x$$

- $$f'(x)=\frac{1}{2\sqrt{|e^{x-2}-3|}}\cdot\frac{|e^{x-2}-3|}{e^{x-2}-3}\cdot e^{x-2}\cdot 1-2=$$

$$D(|x|) = \frac{|x|}{x}$$

$$= \begin{cases} 1 & \text{se } e^{x-2}-3>0 \\ -1 & \text{se } e^{x-2}-3<0 \\ \text{non è definito se } e^{x-2}-3=0 \end{cases}$$

$$= \begin{cases} \dfrac{e^{x-2}}{2\sqrt{e^{x-2}-3}} - 2 & \text{se } e^{x-2}-3>0 \quad e^{x-2}>3 \quad x-2>\log 3 \\ & (x>2+\log 3) \\ -\dfrac{e^{x-2}}{2\sqrt{-e^{x-2}+3}} - 2 & \text{se } e^{x-2}-3<0 \\ & (x<\underbrace{2+\log 3}_{\alpha}) \end{cases}$$

Quando $e^{x-2}-3=0$, cioè $x = 2+\log 3$

si annulla il denom di $f'(x) \Rightarrow x=2+\log 3 \notin \mathrm{dom}(f')$

$\mathrm{dom}(f') = \mathrm{dom}(f) \setminus \{2+\log 3\}$ $\qquad \alpha = 2+\log 3$

$x=2+\log 3$ è un pto di non derivabilità

devo capire se pto ang, cuspide o pto a tg vert.

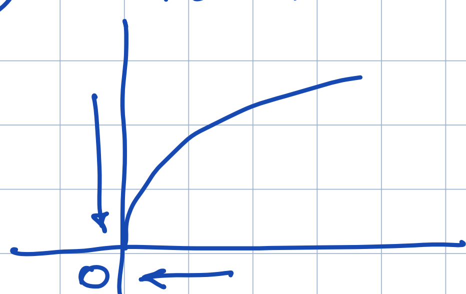
calcolo $f'_-(\alpha)$ e $f'_+(\alpha)$

$$f'_-(\alpha) = \lim_{x\to\alpha^-} f'(x) = \lim_{x\to\alpha^-}\left(\frac{-e^{x-2}}{2\sqrt{-e^{x-2}+3}} - 2 \right) = -\infty$$

($f'_-(\alpha) = \lim_{x\to\alpha^-} f'(x)$) solo se $\exists \lim_{x\to\alpha} f'(x)$

$+$ $\qquad \to 0^+$

se $x \to \alpha = 2+\log 3 \Rightarrow -e^{x-2}+3 \longrightarrow -e^{2+\log 3-2}+3 = 0^+$



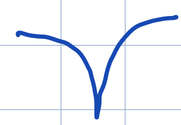
$\exists \lim_{x\to\alpha^-} f'(x) = -\infty \quad \Rightarrow \quad f'_-(\alpha) = \lim_{x\to\alpha^-} f'(x)$

(thm del limite della derivata)

$$f'_+(\alpha) = \lim_{x\to\alpha^+} f'(x) = \lim_{x\to\alpha^+} \frac{\left(e^{x-2}\right)^+}{\underbrace{2\sqrt{\underbrace{e^{x-2}-3}_{\to 0^+}}}_{\to 0^+}} - 2 = +\infty$$

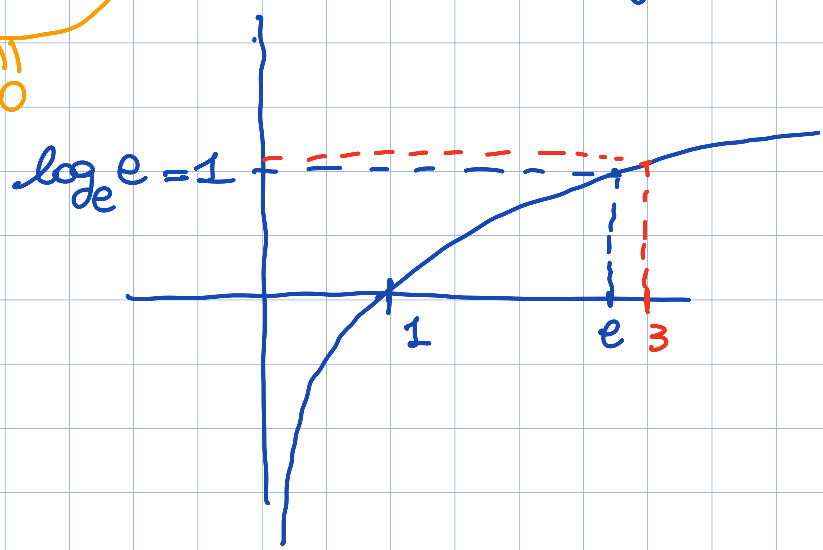
$\Rightarrow f'_-(\alpha) = -\infty \quad f'_+(\alpha) = +\infty$

$x = \alpha = 2+\log 3$ è pto di cuspide

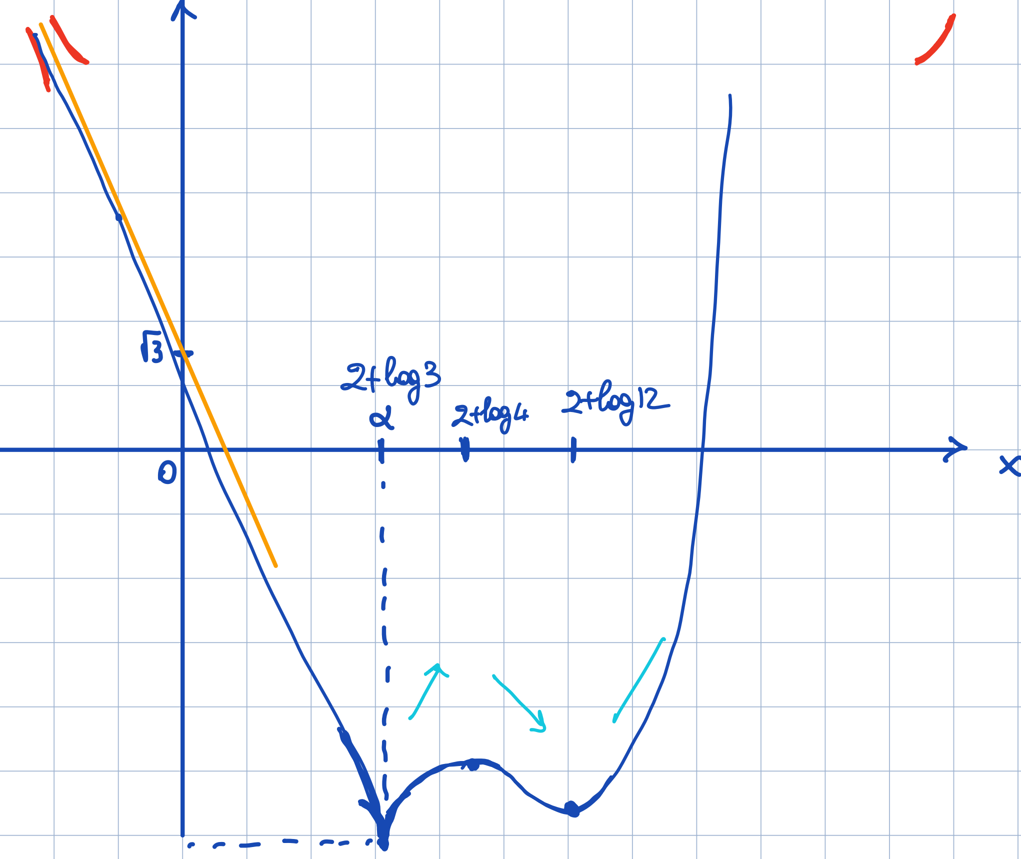


pto di min relativo

$$f(2+\log 3) = \sqrt{\underbrace{|e^{x-2}-3|}_{0}} - 2x = -4-2\log 3 \sim -6$$



$\log 12 = \log(3 \cdot 4) =$
$\log 3 + \log 4 \sim 2.5$



$\alpha = 2 + \log 3 \sim 3$

$y = -2x + \sqrt{3}$

$$f'(x) = \begin{cases} \dfrac{-e^{x-2}}{\sqrt{-e^{x-2}+3}} - 2 & \text{se } x < 2+\log 3 = \alpha \\[2ex] \dfrac{e^{x-2}}{\sqrt{e^{x-2}-3}} - 2 & \text{se } x > 2+\log 3 = \alpha \end{cases}$$

$f'(x) \geq 0$

$\boxed{x < 2 + \log 3}$

$e^{x-2} > 0 \quad \forall x \in \mathbb{R}$

$\sqrt{} > 0 \quad \forall x < 2 + \log 3$

$$\Rightarrow \left.\begin{aligned} -\frac{e^{x-2}}{\sqrt{}} &< 0 \\ -2 &< 0 \end{aligned}\right\} \Rightarrow \text{la somma di 2 numeri } < 0 \text{ è } < 0$$

$\Rightarrow f'(x) < 0 \quad \forall\, x < 2 + \log 3 \Rightarrow f$ è decrescente strett. in $(-\infty, 2 + \log 3)$

$f'(x) \geq 0$ $\qquad$ $f'(x) = \dfrac{e^{x-2}}{2\sqrt{e^{x-2}-3}} - 2 \geq 0$

$\boxed{x > 2 + \log 3}$

$$\frac{e^{x-2} - 4\sqrt{e^{x-2}-3}}{2\sqrt{e^{x-2}-3}} \geq 0$$

$D = 2\sqrt{e^{x-2}-3} > 0$ $\quad \forall\, x > 2 + \log 3$ perché $\sqrt{\ }$ restituisce valori > 0 quando il radicando è > 0

$N:$ $\quad e^{x-2} - 4\sqrt{e^{x-2}-3} \geq 0$

$$\frac{e^{x-2}}{4} \geq \cancel{4}\sqrt{e^{x-2}-3}$$

$$\sqrt{e^{x-2}-3} \leq \frac{e^{x-2}}{4}$$

$\sqrt{A(x)} \leq B(x)$

$$\begin{cases} A(x) \geq 0 & \text{vero perché } x > 2 + \log 3 \\ B(x) \geq 0 & \text{vero} \\ A(x) \leq B^2(x) \end{cases}$$

elevo al quadrato

$$e^{x-2} - 3 \leq \frac{1}{16} e^{2(x-2)}$$

$t = e^{x-2}$

$$t - 3 \leq \frac{1}{16} t^2 \qquad \frac{t^2}{16} - t + 3 \geq 0$$

$$t^2 - 16t + 48 \geq 0$$

$ax^2 + bx + c = 0$

$$x_{1,2} = \frac{-b/2 \pm \sqrt{(b/2)^2 - ac}}{a}$$

$$t_{1,2} = 8 \pm \sqrt{64 - 48} = 8 \pm 4$$

$t_1 = 4 \qquad t_2 = 12$

$t^2 - 16t + 48 \geq 0 \iff t \leq 4$ o $t \geq 12$

$e^{x-2} \leq 4$ o $e^{x-2} \geq 12$

log

$x-2 \le \log 4$ o $x-2 \ge \log 12$

$N \ge 0 \iff \boxed{x \le 2+\log 4 \text{ o } x \ge 2+\log 12}$

		$2+\log 3$		$2+\log 4$		$2+\log 12$	
N	//////		+	0	−	0	+
D	//////		+		+		+
f'	//////		+	0	−	0	+
f	//////		↗		↘		↗

$x = 2+\log 4$ è pto di max rel str.

$x = 2+\log 12$ è pto di min rel str.

f è cresc in $(2+\log 3, 2+\log 4) \cup (2+\log 12, +\infty)$

decr in $(2+\log 4, 2+\log 12)$

Valuto $f(2+\log 4) = \sqrt{\underbrace{e^{\cancel{2}+\log 4-\cancel{2}}}_{4} - 3} - 2(2+\log 4) = 1-4-2\log 4$

$= -3-2\log 4$

~ -5.4

$f(2+\log 12) = \sqrt{e^{\cancel{2}+\log 12-\cancel{2}} - 3} - 2(2+\log 12)$

$= \sqrt{9} - 4 - 2\log 12 = -1-2\log 12 \sim -5.8$

$x = 2+\log 3$ è pto di min assoluto

$\nexists$ pto di max ass perché f è illimitata superiormente

$$f(x) = x + 1 + 2\sqrt{|x| - 1}$$

- $$\lim_{n\to\infty} \frac{(n^{3\alpha} + \log(n^7))\cdot(n-2)!}{n! + 3n}\left(\frac{1}{n} - \sin\frac{1}{n}\right) = (*) \qquad \alpha \in (0, +\infty)$$

- $(n^{3\alpha} + \log n^7) \sim n^{3\alpha}$ per $n \to \infty$
- $n! + 3n \sim n!$ per $n \to \infty$
- $\left(\frac{1}{n} - \sin\frac{1}{n}\right)$ se dicessi $\sin\frac{1}{n} \sim \frac{1}{n}$ per $n \to \infty$

$\left(\qquad \right) \sim 0$ non dà informazioni utili

devo sviluppare $\sin\left(\frac{1}{n}\right)$ con Taylor

$$\sin x = x - \frac{1}{6}x^3 + o(x^4) \Rightarrow \sin\frac{1}{n} = \frac{1}{n} - \frac{1}{6n^3} + o\left(\frac{1}{n^4}\right)$$

per $x \to 0$ $\qquad$ per $n \to \infty$

$$\frac{1}{n} - \sin\frac{1}{n} = \cancel{\frac{1}{n}} - \cancel{\frac{1}{n}} + \frac{1}{6n^3} + o\left(\frac{1}{n^4}\right) = \frac{1}{6n^3} + o\left(\frac{1}{n^4}\right) \sim \frac{1}{6n^3}$$

$$(*) = \lim_{n\to\infty} \frac{n^{3\alpha}\,\cancel{(n-2)!}}{\underbrace{n!}_{n\underbrace{(n-1)}_{\sim n}\cancel{(n-2)!}}} \cdot \frac{1}{6n^3} = \lim_{n\to\infty} \frac{n^{3\alpha}}{6n^5} = \frac{1}{6}\lim_{n\to\infty} n^{3\alpha-5} =$$

$n! > (n-2)!$ sviluppo $n!$

$$n! = n(n-1)(n-2)!$$

$$= \begin{cases} +\infty & \text{se } 3\alpha-5>0 \iff \alpha > 5/3 \\ \frac{1}{6} & \text{se } 3\alpha-5=0 \iff \alpha = 5/3 \\ 0 & \text{se } 3\alpha-5<0 \iff 0<\alpha<\frac{5}{3} \end{cases}$$

$$\lim_{x\to 0^+} \frac{(x-\sin x)^2 + \log(1+x^8) - \frac{1}{36}\operatorname{tg} x^6}{(\operatorname{aTan} x)^\alpha} \quad \circledast \qquad \alpha \in \mathbb{R}$$

$$\operatorname{tg} t = t + \frac{t^3}{3} + o(t^4)$$

$$\log(1+t) = t - \frac{t^2}{2} + o(t^2)$$

$$\sin t = t - \frac{t^3}{6} + o(t^4)$$

$$N: \ (x-\sin x)^2 = \left(\cancel{x}-\cancel{x}+\frac{1}{6}x^3+o(x^4)\right)^2$$

$$= \frac{1}{36}x^6 + o(x^8) + \underline{o(x^7)}$$

doppio prodotto

$$\log(1+x^8) = x^8 - \frac{x^{16}}{2} + o(x^{16})$$

$$\frac{1}{36}\operatorname{tg} x^6 = \frac{1}{36}x^6 + \frac{1}{3\cdot 36}x^{18} + o(x^{24})$$

$$N = \cancel{\frac{1}{36}x^6} + \underline{o(x^7)} + \underline{x^8} - \underbrace{\frac{x^{16}}{2} + o(x^{16})} - \cancel{\frac{1}{36}x^6} - \underbrace{\frac{1}{3\cdot 36}x^{18} + o(x^{24})}_{o(x^9)}$$

$$= o(x^7) \quad \text{troppo inaccurato}$$

devo prendere $\sin x = x - \frac{x^3}{6} + \frac{x^5}{5!} + o(x^6)$

$$(x-\sin x)^2 = \left(\frac{x^3}{6} - \frac{x^5}{5!} + o(x^6)\right)^2$$

$$= \frac{x^6}{36} - \frac{2}{6\cdot 5!}x^8 + o(x^9)$$

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$$

$$N = \cancel{\frac{1}{36}x^6} - \frac{2}{6 \cdot 5!}x^8 + o(x^9) + x^8 - \cancel{\frac{1}{36}x^6}$$ posso trascurare

$$= \left(1 - \frac{2}{6 \cdot 5!}\right)x^8 + o(x^9)$$

D : $\arctan x = x + o(x) \Rightarrow (\arctan x)^\alpha = x^\alpha + o(x^\alpha)$

In conclusione ⊛ $\lim_{x \to 0^+} \frac{\frac{359}{360} \cdot x^8}{x^\alpha} = \frac{359}{360} \lim_{x \to 0^+} x^{8-\alpha} =$

$$= \begin{cases} 0 & \text{se } 8-\alpha > 0 \quad \alpha < 8 \\ 359/360 & \text{se } 8-\alpha = 0 \quad \alpha = 8 \\ +\infty & \text{se } 8-\alpha < 0 \quad \alpha > 8 \end{cases}$$