Polinomi di Taylor

$$P_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k =$$

$$\mathbb{R} \ni$$

$$= \underbrace{\frac{f^{(0)}(x_0)}{0!}(x-x_0)^0}_{k=0} + \underbrace{\frac{f^{(1)}(x_0)}{1!}(x-x_0)^1}_{k=1} + \underbrace{\frac{f^{(2)}(x_0)}{2!}(x-x_0)^2}_{k=2} + \dots + \underbrace{\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n}_{k=n}$$

$\in \mathbb{R}$ $\quad \in \mathbb{R}$ $\quad \in \mathbb{R}$

prendo $f(x) = e^x$, $x_0 = 0$ $n = 3$, ? $P_3(x)$ pol di Taylor

$$f(x) = e^x \qquad f(0) = 1$$
$$f'(x) = e^x \qquad f'(0) = 1$$
$$f''(x) = e^x \qquad f''(0) = 1$$
$$f'''(x) = e^x \qquad f'''(0) = 1$$

$$P_3(x) = \underbrace{\frac{1}{1}\underbrace{(x-0)^0}_{1}}_{k=0} + \underbrace{\frac{1}{1}(x-0)}_{k=1} + \underbrace{\frac{1}{2}(x-0)^2}_{k=2} + \underbrace{\frac{1}{6}(x-0)^3}_{k=3}$$

$$= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$$

pol di Taylor di grado 3 che appx $f(x) = e^x$ in un intorno di $x_0 = 0$

$$= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3$$

$$r_3(x) = o\left((x-x_0)^3\right) = o(x^3)$$

$$\boxed{f(x) = e^x = P_3(x) + o(x^3) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + o(x^3)}$$

$$e^x = P_4(x) + o(x^4)$$

$$P_4(x) = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4$$

Calcolo $P_3(x)$ di Taylor che approssima $f(x)=\sin x$ in $I(x_0)$ con $x_0=0$

$$P_3(x)=\sum_{k=0}^{3}\frac{f^{(k)}(0)}{k!}\cdot x^k=\frac{f^{(0)}(0)}{0!}x^0+\frac{f^{(1)}(0)}{1!}x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3$$

$f(x)=\sin x \qquad f(0)=0 \qquad f^{IV}(x)=+\sin x$

$f'(x)=\cos x \qquad f'(0)=1 \qquad f'''(x)=+\cos x$

$f''(x)=-\sin x \qquad f''(0)=0$

$f'''(x)=-\cos x \qquad f'''(0)=-1$

$$P_3(x)=\underbrace{\cancel{\frac{0}{1}\cdot 1}}_{k=0}+\underbrace{\frac{1}{1}\cdot x}_{k=1}+\underbrace{\cancel{\frac{0}{2}\cdot x^2}}_{k=2}+\frac{-1}{3!}x^3=x-\frac{1}{3!}x^3$$

$$P_4(x)=\underbrace{x-\frac{1}{3}x^3}_{P_3(x)}+\frac{\overbrace{f^{(4)}(0)}^{0}}{4!}x^4=P_3(x)$$

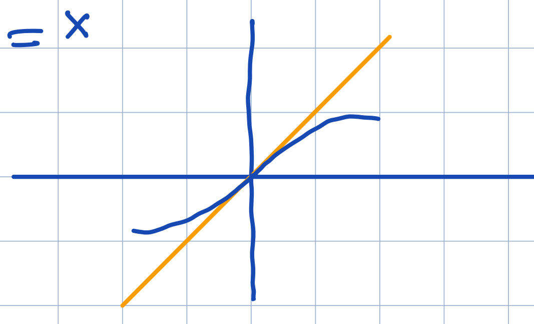
$$P_5(x)=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5$$

$$\sin x=P_3(x)+r_3(x)=x-\frac{1}{3}x^3+\underbrace{o(x^3)}_{x^4,\ x^5,\ \dots}$$

$$=x-\frac{1}{3}x^3+o(x^4)$$

so che la prima potenza esclusa è $x^5=o(x^4)$

? $$P_1(x)=\underbrace{\frac{f^{(0)}(0)}{0!}x^0}_{0}+\underbrace{\frac{f'(0)}{1}x}_{x}=x$$



$$\boxed{\sin x \underset{0}{=} x+o(x)}$$

o sotto: 0

$$\lim_{x\to 0} \frac{x - \sin x}{x^3} = \frac{0}{0} \qquad \sin x = x - \frac{1}{3!}x^3 + o(x^3)$$

$$= \lim_{x\to 0} \frac{x - \left(x - \frac{1}{6}x^3 + o(x^3)\right)}{x^3} = \lim_{x\to 0} \frac{\cancel{x} - \cancel{x} + \frac{1}{6}x^3 + o(x^3)}{x^3}$$

$$= \lim_{x\to 0} \frac{\frac{1}{6}x^3}{x^3} = \frac{1}{6}$$

se $x \to 0$ $\quad \sin x \sim x$

$$\lim_{x\to 0} \frac{x - \sin x}{x^3} = \lim_{x\to 0} \frac{\overset{0}{x - x}}{x^3} = 0 \quad \| \quad \text{NO}$$

$\sin x \sim x$

Se io parto da $\lim_{x\to 0} \frac{x-x}{x^3} = \lim_{x\to 0} \frac{0}{x^3} = 0$

$\uparrow \neq 0$

$$\lim_{x\to 0} \frac{x - \sin x}{x^3} = \lim_{x\to 0} \frac{x - (x + o(x))}{x^3} = \lim_{x\to 0} \frac{o(x)}{x^3} \quad \| \quad \text{corretto}$$

ma non dà risposta