$f(x) = 2\sin x + \cos(2x)$

- $\mathrm{dom}(f) = \mathbb{R}$
- simmetrie $\quad f(-x) = 2\cdot\sin(-x) + \cos(-2x) =$

$$= -2\sin(x) + \cos(2x) \neq f(x) \Rightarrow f \text{ non è pari}$$
$$\neq -f(x) \Rightarrow f \text{ non è disp.}$$

non ho simmetrie rispetto a 0.

- verifico che $f(x)$ è periodica con periodo $T = 2\pi$

se $f(x+2\pi) = f(x) \quad \forall x \in \mathrm{dom}(f) \Rightarrow f$ è periodica con $T = 2\pi$

$$f(x+2\pi) = 2\sin(x+2\pi) + \cos(\underbrace{2(x+2\pi)}_{2x+4\pi}) =$$

$$= 2\sin(x) + \cos(2x) = f(x)$$

per la periodicità pari a 2π di $\sin x$ e $\cos x$

- poiché f è periodica $\Rightarrow \nexists \lim_{x\to\pm\infty} f(x) \Rightarrow \nexists$ as oriz/obl

$\nexists$ as vert perché $\mathrm{dom}(f) = \mathbb{R}$, non sono esclusi pti finiti dal dom

limito lo studio a $[0, 2\pi]$

$f(x) = 2\sin x + \cos(2x)$

- $f'(x) = 2\cdot\cos x - \sin(2x)\cdot 2 =$

$= 2\cos x - 4\sin x\cdot\cos x$

$= 2\cos x\cdot(1 - 2\sin x)$

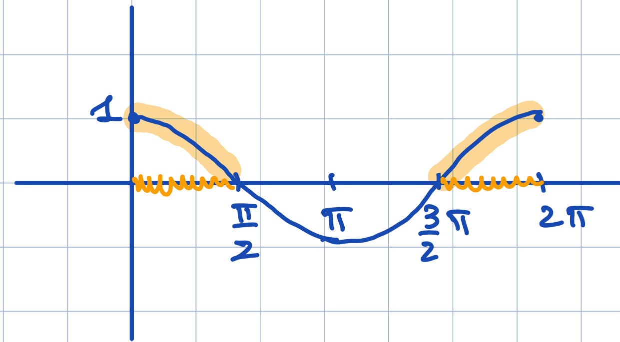
($\sin(2x) = 2\sin x\cdot\cos x$

- pti stazionari $\equiv f'(x) = 0$

$2\cos x\cdot(1-2\sin x) = 0$ quando $\cos x = 0$ o $1 - 2\sin x = 0$

$\cos x = 0 \iff x = \frac{\pi}{2}$ o $x = \frac{3}{2}\pi$

sono pti staz



$1 - 2\sin x = 0$

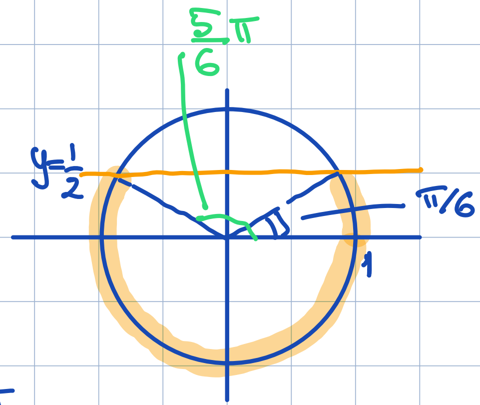
$2\sin x = 1$

$\sin x = \frac{1}{2}$

$\Updownarrow$

$x = \frac{\pi}{6}$ o $x = \frac{5}{6}\pi$

punti staz.



- $f'(x) \geq 0 \iff 2\cos x \cdot (1 - 2\sin x) \geq 0$

$F_1 = \cos x \geq 0 \iff 0 \leq x \leq \frac{\pi}{2} \cup \frac{3\pi}{2} \leq x \leq 2\pi$

$F_2 = 1 - 2\sin x \geq 0 \iff \sin x \leq \frac{1}{2} \iff$

$0 \leq x \leq \frac{\pi}{6} \cup \frac{5}{6}\pi \leq x \leq 2\pi$

	0		$\pi/6$		$\frac{\pi}{2}$		$\frac{5}{6}\pi$		$\frac{3}{2}\pi$		2π
F_1		+		+	0	−		−	0	+	
F_2		+	0	−		−	0	+		+	
f'		+	0	−	0	+	0	−	0	+	
f		↗		↘		↗		↘		↗	

$f(0) = 1$

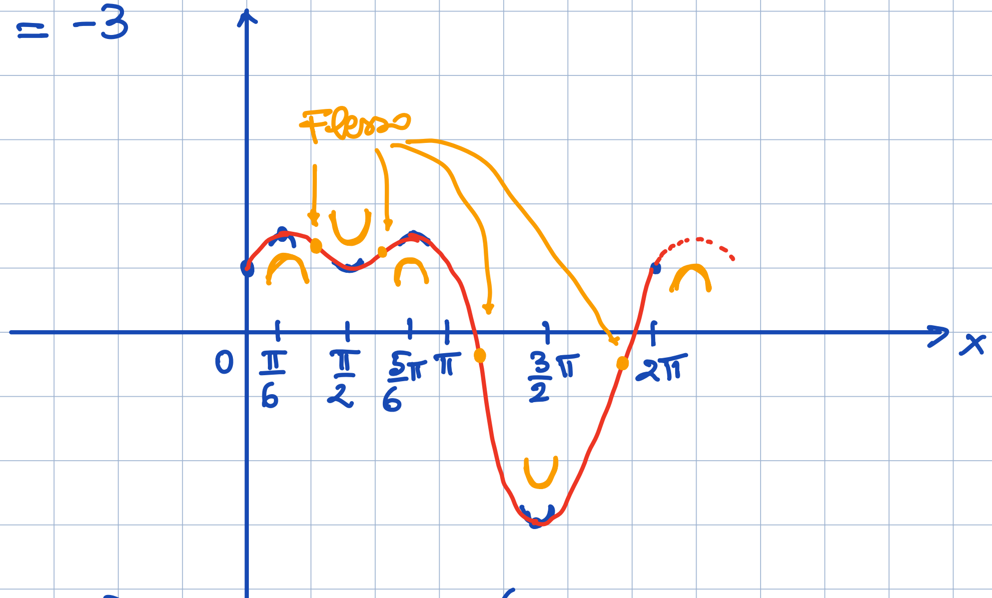
$f\left(\frac{\pi}{6}\right) = 2 \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

$f\left(\frac{\pi}{2}\right) = 2 - 1 = 1$

$f\left(\frac{5}{6}\pi\right) = 2 \cdot \frac{1}{2} + \frac{1}{2} = \frac{3}{2}$

$f(x) = 2\sin x + \cos(2x)$

$f\left(\frac{3}{2}\pi\right) = -2 - 1 = -3$



f è crescente in $\left(0,\frac{\pi}{6}\right) \cup \left(\frac{\pi}{2},\frac{5}{6}\pi\right) \cup \left(\frac{3}{2}\pi, 2\pi\right)$

decrescente in $\left(\frac{\pi}{6},\frac{\pi}{2}\right) \cup \left(\frac{5}{6}\pi,\frac{3}{2}\pi\right)$

$x=\frac{\pi}{6}$ e $x=\frac{5}{6}\pi$ sono pti di max rel e ass, e pti staz

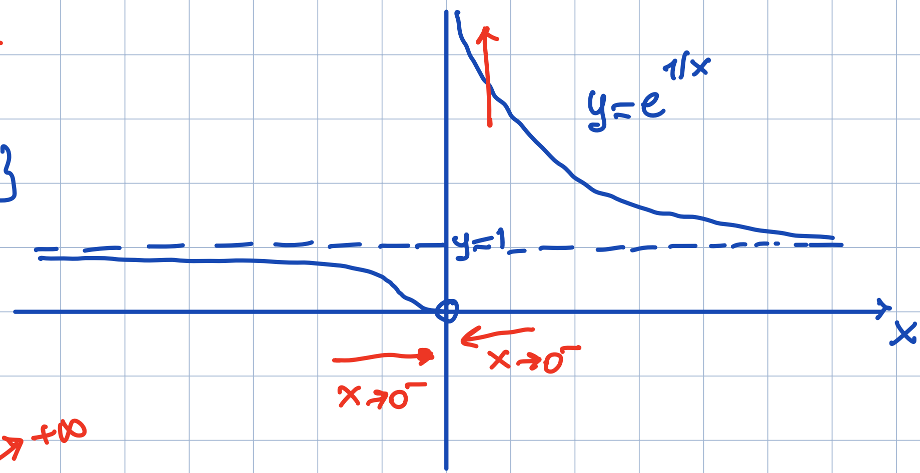
$x=\frac{\pi}{2}$ è pto di min rel staz

$x=\frac{3}{2}\pi$ è pto di min rel e ass, staz.

$\exists$ 4 punti flesso in $[0, 2\pi]$ –

$$f(x) = e^{1/x}$$

$$\text{dom}(f) = \mathbb{R} \setminus \{0\}$$



$$\lim_{x\to 0^+} \frac{\overset{\to +\infty}{e^{1/x}}}{\underset{\to +\infty}{\frac{1}{x^4}}} = +\infty \quad \text{per confronto di infiniti}$$

$$t = \frac{1}{x} \qquad \lim_{x\to 0^+} \frac{e^{1/x}}{\frac{1}{x^4}} = \lim_{t\to +\infty} \frac{e^t}{t^4} \overset{(H)}{=} \lim_{t\to +\infty} \frac{e^t}{4t^3} = \frac{\infty}{\infty}$$

$$\overset{(H)}{=} \lim_{t\to +\infty} \frac{e^t}{12t^2} = \frac{\infty}{\infty}$$

$$\overset{(H)}{=} \lim_{t\to +\infty} \frac{e^t}{24t} \overset{(H)}{=} \lim_{t\to \infty} \frac{e^t}{24} = +\infty$$

$$\exists$$

$$\Rightarrow \lim_{x\to 0^+} \frac{e^{1/x}}{1/x^4} = +\infty$$

$$\lim_{x\to 0^-} \frac{e^{\overset{\to -\infty}{1/x}}}{x^4} = \frac{0}{0} \qquad t = -\frac{1}{x} \quad \Rightarrow \quad \frac{1}{x} = -t$$

$$x = -\frac{1}{t}$$

$$= \lim_{t\to +\infty} \frac{e^{-t}}{\left(-\frac{1}{t}\right)^4} = \lim_{t\to +\infty} \frac{t^4}{e^t} =$$

$$= \dots \overset{(H)}{=} \lim_{t\to +\infty} \frac{24}{e^t} = 0$$

$$\Rightarrow \lim_{x\to 0^-} \frac{e^{1/x}}{x^4} = 0$$ cioè $e^{1/x} \to 0$ più veloce di x^4 quando $x \to 0^-$

$$\lim_{x \to +\infty} \frac{\log x}{x} = 0 \qquad \lim_{x \to +\infty} \frac{(\log x)^\alpha}{x^\beta} = 0 \quad \forall \alpha, \beta > 0$$

es: $$\lim_{x \to +\infty} \frac{(\log x)^4}{\sqrt{x}} = 0$$

Una qualsiasi potenza positiva di x vince su una qualsiasi potenza positiva del $\log x$ quando $x \to +\infty$

$$(\log x)^4 = \log^4 x \neq \log x^4 = 4 \log x$$

$$\log x^4 = \log(x^4)$$

$$\lim_{x \to 0^+} x \cdot \log x = 0 \qquad \lim_{x \to 0^+} x^\alpha (\log x)^\beta = 0 \quad \forall \alpha, \beta > 0$$

$$\lim_{x \to 0^+} x^2 \cdot \log x = 0$$

$$\lim_{x \to 0^+} \sqrt[3]{x} \cdot (\log x)^3 = 0$$

$$f(x) = e^{|\log x| - \frac{1}{3x}}$$

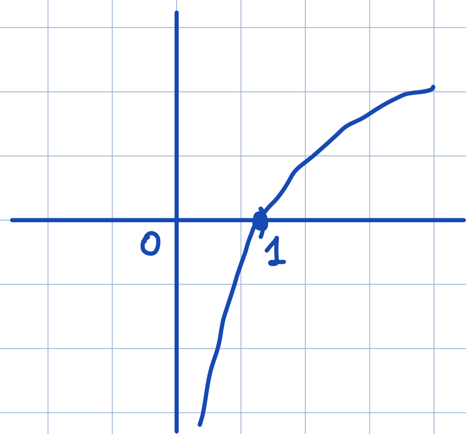
- dom (f) $\quad \begin{cases} x > 0 & \text{per il log} \\ x \neq 0 \end{cases} \Rightarrow$ dom $(f) = (0, +\infty)$

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

$$\boxed{|g(x)| = \begin{cases} g(x) & \text{se } g(x) \geq 0 \\ -g(x) & \text{se } g(x) < 0 \end{cases}}$$

$$e^{|\log x|} = \begin{cases} e^{\log x} & \text{se } \log x \geq 0 \\ e^{-\log x} & \text{se } \log x < 0 \end{cases}$$

$$= \begin{cases} x & \text{se } x \geq 1 \\ \frac{1}{x} & \text{se } 0 < x < 1 \end{cases}$$



$$e^{-\log x} = e^{\log(x^{-1})} = e^{\log \frac{1}{x}} = \frac{1}{x}$$

$$\Rightarrow f(x) = e^{|\log x| - \frac{1}{3x}} = e^{|\log x|} \cdot e^{-\frac{1}{3x}} =$$

$$f(x) = \begin{cases} x\, e^{-1/3x} & \text{se } x \geq 1 \\ \frac{1}{x} e^{-\frac{1}{3x}} & \text{se } 0 < x < 1 \end{cases}$$

dom $(f) = (0, +\infty)$

- simmetrie non esistono perché dom (f) non è sim.
- limiti.

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{1}{\underbrace{x}_{\to 0^+}} \underbrace{e^{-1/3x}}_{\to 0} = \frac{0}{0}$$

$-\frac{1}{3x} \to -\infty$

$$t = \frac{1}{3x}$$

$$x = \frac{1}{3t}$$

$$= \lim_{t\to+\infty} \frac{e^{-t}}{\frac{1}{3t}} =$$

$$= \lim_{t\to+\infty} \frac{3t}{e^{t}} = 0^+$$

(con de l'Hopital o confronto di ∞)

$\lim_{x\to0^+} f(x) = 0^+ \Rightarrow \nexists$ as vert in $x=0$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \underbrace{x}_{+\infty}\, \underbrace{e^{\overbrace{-\frac{1}{3x}}^{\to 0}}}_{\to 1} = +\infty$$

potrebbe $\exists$ as. obliquo, calcolo m e q

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\, e^{-1/3x}}{\cancel{x}} = 1 \qquad m = 1 \in \mathbb{R}$$

$$q = \lim_{x\to+\infty} (f(x) - mx) = \lim_{x\to+\infty} \left(x e^{-\frac{1}{3x}} - x\right) =$$

$$= \lim_{x\to+\infty} \underbrace{x}_{+\infty} \cdot \underbrace{\left(e^{-\frac{1}{3x}} - 1\right)}_{0}$$

osservo che $\underbrace{-\frac{1}{3x}}_{t} \to 0$ quando $x \to +\infty$

e ricordo che $\lim_{t\to0} \frac{e^t - 1}{t} = 1$ cioè

$e^t - 1 \sim t$ quando $t \to 0$

$$\Rightarrow e^{\overbrace{-\frac{1}{3x}}^{t\to0}} - 1 \sim \overbrace{-\frac{1}{3x}}^{t} \quad \text{quando } x \to +\infty$$

$$= \lim_{x\to+\infty} \cancel{x} \left(-\frac{1}{3\cancel{x}}\right) = -\frac{1}{3} = q$$

$$\Rightarrow \quad y = x - \frac{1}{3} \text{ è as. obl dx}$$

- studio in $x = 1$

$$\lim_{x\to 1^-} f(x) = \lim_{x \to 1^-} \overset{\to 1}{x}\, e^{-\overset{\to \frac{1}{3}}{1/3x}} = e^{-1/3}$$

$$\lim_{x\to 1^+} f(x) = \lim_{x \to 1^+} \frac{1}{x} e^{-1/3x} = e^{-1/3}$$

$$f(1) = e^{-1/3}$$

$\Rightarrow$ f è cont in $x=1$ e vale $e^{-1/3}$

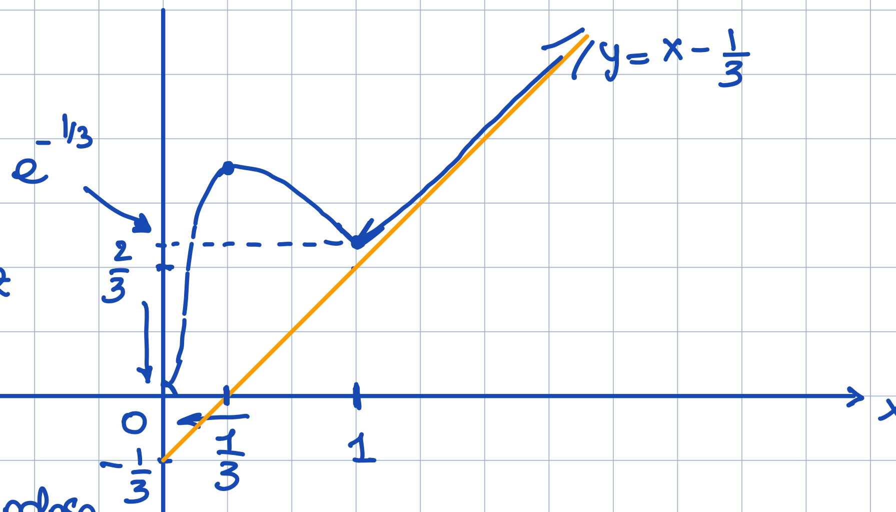
f è cresc in $\left(0, \frac{1}{3}\right) \cup (1, +\infty)$

decresc in $\left(\frac{1}{3}, 1\right)$

$x = \frac{1}{3}$ è pto di max rel stretto

$\nexists$ pto di max ass perché f è illimitata

$x = 1$ è pto di min rel angoloso



$x = 0$ non è pto di min perché $x = 0 \notin \text{dom} f$, $0 = \inf(\text{im}(f))$

$$f(x) = \begin{cases} x \cdot e^{-1/3x} & \text{se } x \geq 1 \\ \frac{1}{x} e^{-\frac{1}{3x}} & \text{se } 0 < x < 1 \end{cases}$$

$$f'(x) = \begin{cases} e^{-\frac{1}{3x}}\left(1 + \frac{1}{3x}\right) & \text{se } x > 1 \\ \frac{1}{3x^3} e^{-\frac{1}{3x}}(-3x+1) & \text{se } x < 1 \end{cases}$$

$$D\left(x \cdot e^{-1/3x}\right) = 1 \cdot e^{-\frac{1}{3x}} + x \cdot D\left(e^{-1/3x}\right) = e^{-\frac{1}{3x}} + \cancel{x} \frac{1}{3x^{\cancel{2}}} \cdot e^{-\frac{1}{3x}} =$$

$$D\left(e^{-\frac{1}{3x}}\right) = e^{-\frac{1}{3x}} \cdot \left(-\frac{1}{3} \cdot -\frac{1}{x^2}\right) = \frac{1}{3x^2} \cdot e^{-\frac{1}{3x}}$$

$$D\left(\frac{1}{x}\right)=\frac{-1}{x^2}$$

$$= e^{-\frac{1}{3x}}\left(1+\frac{1}{3x}\right)$$

$$D\left(\frac{1}{x}\cdot e^{-1/3x}\right)=\left(-\frac{1}{x^2}\right)\cdot e^{-\frac{1}{3x}}+\frac{1}{x}\cdot D\left(e^{-1/3x}\right)=$$

$$=-\frac{1}{x^2}e^{-\frac{1}{3x}}+\frac{1}{x}\cdot\frac{1}{3x^2}e^{-1/3x}=$$

$$=\frac{1}{3x^3}e^{-\frac{1}{3x}}\left(-3x+1\right)$$

$$f'(x)=\begin{cases} e^{-\frac{1}{3x}}\left(1+\frac{1}{3x}\right) & \text{se } x>1 \\ \frac{1}{3x^3}e^{-\frac{1}{3x}}\left(-3x+1\right) & \text{se } x<1 \end{cases}$$

? in $x=1$

$$f'_-(1) \underset{def}{=} \lim_{x\to 1^-}\frac{f(x)-f(1)}{x-1} \overset{?}{=} \lim_{x\to 1^-} f'(x)$$

Calcolo $\lim_{x\to 1^-} f'(x) = \lim_{x\to 1^-}\frac{1}{3x^3}e^{-\frac{1}{3x}}(-3x+1) = -\frac{2}{3}e^{-\frac{1}{3}}$.

$x^3 \to 1$, $\frac{1}{3x}\to\frac{1}{3}$, $x \to 1$

$\Rightarrow$ per il thm del limite della derivata

$$f'_-(1)=\lim_{x\to 1^-} f'(x) = -\frac{2}{3}e^{-1/3}$$

$$f'_+(1) \overset{?}{=} \lim_{x\to 1^+} f'(x) = \lim_{x\to 1^+} e^{-\frac{1}{3x}}\left(1+\frac{1}{3x}\right) = \frac{4}{3}e^{-\frac{1}{3}}$$

$$-\frac{2}{3}e^{-\frac{1}{3}} \neq \frac{4}{3}e^{-\frac{1}{3}}$$

$f'_-(1) \neq f'_+(1)$ sono entrambi finiti $\Rightarrow$

$x=1$ pto angoloso

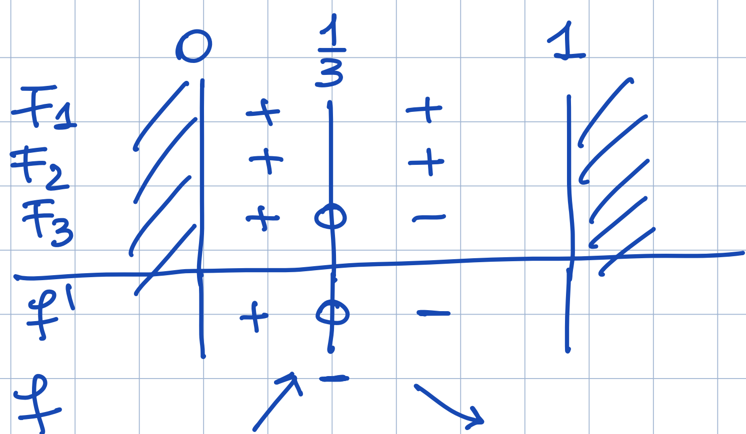
- $f'(x) \geq 0$

$\boxed{0<x<1}$ $\quad f'(x) = \frac{1}{3x^3} e^{-\frac{1}{3x}}(-3x+1) \geq 0$

$F_1 = \frac{1}{3x^3} \geq 0 \quad \forall x>0$ e quindi nell'intervallo considerato $\forall x : 0<x<1$

$F_2 = e^{-\frac{1}{3x}} \geq 0 \quad \forall x\neq 0$ e quindi sempre positiva in $(0,1)$

$F_3 = (-3x+1) \geq 0 \iff x \leq \frac{1}{3}$

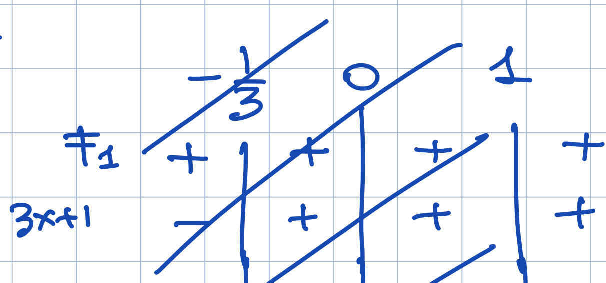


$\boxed{x>1}$ $\quad f'(x) = e^{-\frac{1}{3x}}\left(1+\frac{1}{3x}\right) \geq 0$

$F_1 = e^{-1/3x} > 0 \quad \forall x\neq 0 \Rightarrow$ anche in $(1,+\infty)$

$F_2 = \underbrace{1+\frac{1}{3x}}_{>0 \ \forall x>1} = \frac{3x+1}{3x}$ $\qquad 3x+1 \geq 0 \qquad x \geq -\frac{1}{3}$

$\qquad x>0 \qquad x>0$



x	$-$	$-$	$+$	$+$
f'				$+$
f				$\nearrow$