• Area sottesa al grafico di

$$f(x) = \frac{1}{x} \cdot \frac{1}{(\log^2 x + 4\log x + 5)}$$

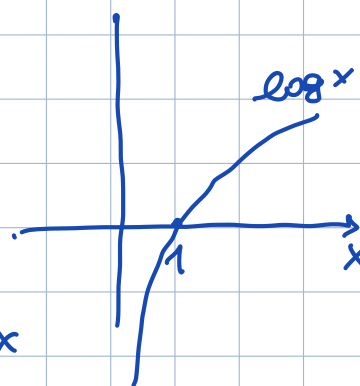
nell'intervallo $[1, e]$

$$A = \int_1^e |f(x)|\,dx = \int_1^e \frac{1}{|x \cdot (\log^2 x + 4\log x + 5)|}\,dx$$

Analizzo il contenuto del modulo per $1 \leq x \leq e$

$x > 0$ in $[1, e]$

? $\underbrace{\underbrace{\log^2 x}_{\geq 0} + 4\underbrace{\log x}_{\geq 0} + 5}_{\geq 5}$ se $x \in [1, e] \Rightarrow \log x \geq 0$



$\Rightarrow |f(x)| = f(x) \quad \forall\, x \in [1, e]$

e $A = \int_1^e f(x)\,dx = \int_1^e \frac{1}{x(\log^2 x + 4\log x + 5)}\,dx$

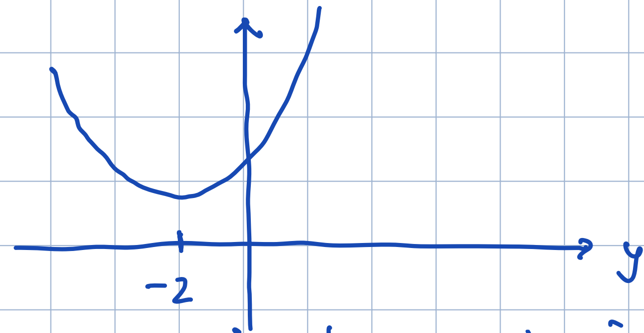
Cerco una primitiva $G(x)$ di $f(x)$

$$G(x) = \int \underbrace{\frac{1}{x(\log^2 x + 4\log x + 5)}\,dx}_{=\,dy} =$$

$y = \log x \qquad dy = \frac{1}{x}\,dx$

$$= \int \frac{dy}{y^2 + 4y + 5}$$ funz. fratta

? $y^2+4y+5=0$ $\Delta = b^2-4ac = 16-20<0$



∄ radici
non posso scomporre il trinomio in un prodotto di 2 fattori.

$\longrightarrow t^2+1$

$$y^2+4y+5 = (y^2+4y+4)+1 = (y+2)^2+1$$

altra sost $t = y+2$ $dt = dy$

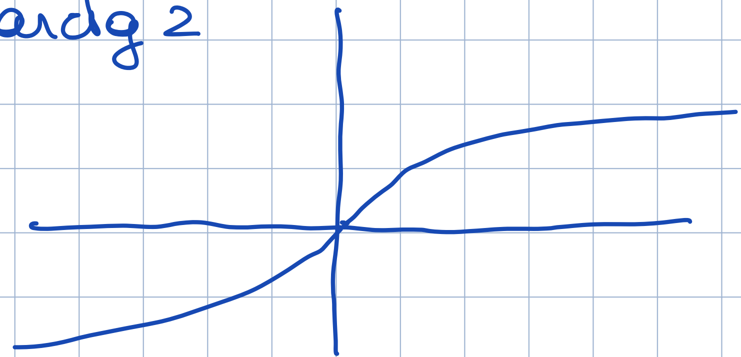
$$\int \frac{1}{y^2+4y+5}\,dy = \int \frac{1}{(y+2)^2+1}\,dy = \int \frac{1}{t^2+1}\,dt =$$

$$= \operatorname{arctg}(t) = \operatorname{arctg}(y+2) = \operatorname{arctg}(\log x + 2) = G(x)$$

$$A = \int_1^e f(x)\,dx = \left[G(x) \right]_1^e = G(e) - G(1) =$$

$$= \operatorname{arctg}(\underbrace{\log e}_{1} + 2) - \operatorname{arctg}(\underbrace{\log 1}_{0} + 2)$$

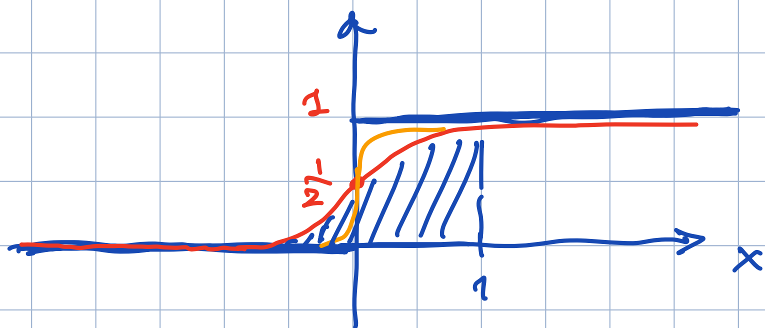
$$= \operatorname{arctg} 3 - \operatorname{arctg} 2$$



• $\int_{-\infty}^{1} \frac{1}{1+e^{-x}}\,dx$

$$\int_{-\infty}^{a} f(x)\,dx = \lim_{x\to-\infty} \int_{x}^{a} f(t)\,dt$$

$$f(x) = \frac{1}{1+e^{-x}} \qquad \text{sigmoide}$$



$$\int_{-\infty}^{1} \frac{1}{1+e^{-x}}\,dx = \lim_{x\to-\infty} \boxed{\int_{x}^{1} \frac{1}{1+e^{-t}}\,dt} \overset{(*)}{=}$$

integrale definito

Cerco una primitiva $G(t)$

$$G(t) = \int \frac{1}{1+e^{-t}}\,dt = \int \frac{1}{e^{-t}(e^{t}+1)}\,dt = \int \frac{\overbrace{e^{t}\,dt}^{dy}}{\underbrace{e^{t}+1}_{y}} =$$

$$y = e^{t}+1 \qquad dy = e^{t}\cdot dt$$

$$= \int \frac{dy}{y} = \log|y| = \log(e^{t}+1)$$

$\overset{(*)}{=} \lim_{x\to-\infty} \left[\log(e^t+1)\right]_x^1 = \lim_{x\to-\infty} \left(\log(e+1) - \underbrace{\log(e^x+1)}_{0}\right)$

$= \log(e+1)$

integrale conv.

• $\displaystyle\int_{-\frac{\pi}{3}}^{\pi/3} (\cos^3 x - \sin^3 x)\,dx =$

se f è dis $\Rightarrow \displaystyle\int_{-a}^{a} f(x)\,dx = 0$

se f è pari $\Rightarrow \displaystyle\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

$= \displaystyle\int_{-\frac{\pi}{3}}^{\pi/3} \cos^3 x\,dx - \underbrace{\int_{-\frac{\pi}{3}}^{\pi/3} \sin^3 x\,dx}_{=0} =$

$g(x) = \cos^3 x$ è pari

$g(-x) = \cos^3(-x) = [\cos(-x)]^3 = \cos^3(x) = g(x)$

cos è pari

$g(x) = \sin^3(x)$ dispari

$g(-x) = \sin^3(-x) = [\sin(-x)]^3 = [-\sin(x)]^3 =$

$= -\sin^3 x = -g(x)$

$= 2\displaystyle\int_0^{\pi/3} \cos^3 x\,dx = 2\int_0^{\pi/3} \cos x \cdot \underbrace{\cos^2 x}_{(1-\sin^2 x)} \underbrace{dx}_{dy} =$

$y = \sin x \quad dy = \cos x \cdot dx$

se $x=0 \Rightarrow y=0$, se $x=\pi/3 \Rightarrow y = \sqrt{3}/2$

$$=2\int_0^{\sqrt{3}/2} (1-y^2)\,dy = 2\left[y-\frac{1}{3}y^3\right]_0^{\sqrt{3}/2} =$$

primitiva $G(y) = y - \frac{1}{3}y^3$

$$= 2\left[\frac{\sqrt{3}}{2} - \frac{1}{\cancel{3}}\,\frac{\cancel{3}\sqrt{3}}{8} - 0\right] = \sqrt{3} - \frac{\sqrt{3}}{4} = \sqrt{3}\cdot\frac{3}{4}$$

- Calcolare l'area della regione di piano limitata dalle 2 curve di eqz

$$f(x) = \frac{4}{4-x} \quad e \quad g(x) = (x-1)^2$$

$$\left(\frac{N}{D}\right)' = \frac{N'\cdot D - N\cdot D'}{D^2}$$

$f(x) = \frac{4}{4-x}$ dom $(f) = \mathbb{R}\setminus\{4\}$

$\lim_{x\to 4^{\pm}} \frac{4}{4-x} = \mp\infty$ $x=4$ as. vert.

$\lim_{x\to\pm\infty} \frac{4}{4-x} = 0$ $y=0$ as. orizz.

$$f'(x) = \frac{0-4\cdot(-1)}{(4-x)^2} = \frac{4}{(4-x)^2} \rightarrow f'(x) \geq 0 \ \forall x \in \text{dom}(f)$$

f è crescente su tutto il dominio

$$f''(x) = \frac{0-4\cdot 2(4-x)\cdot(-1)}{(4-x)^4} = \frac{8\,\cancel{(-x+4)}}{(4-x)^{\cancel{4}\,3}} = \frac{8}{(4-x)^3}$$

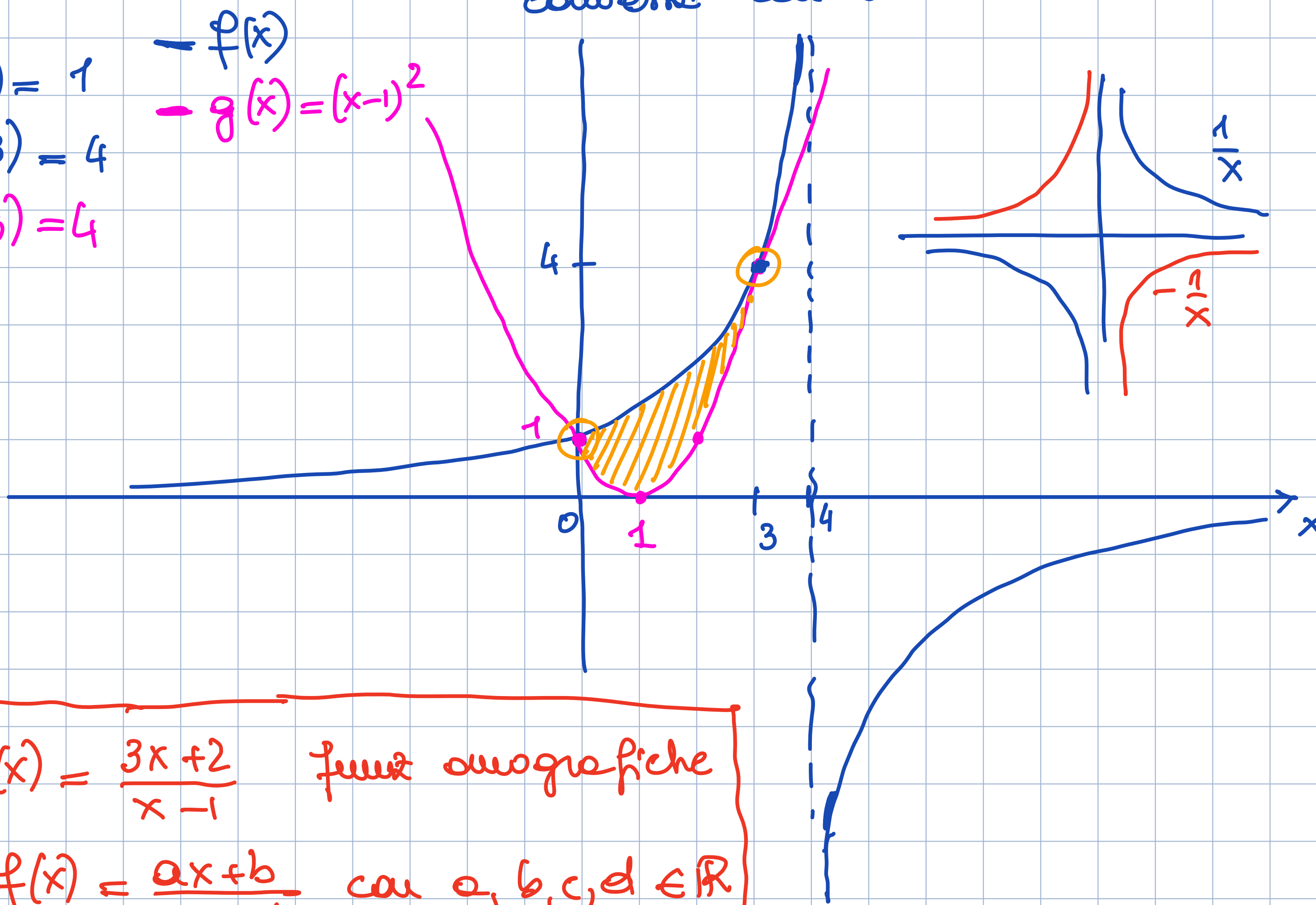
$$f''(x) \geq 0 \iff (4-x) > 0 \iff x < 4$$

f''	$+$	4	$-$
f	$\cup$		$\cap$
	convessa		concava

$f(0) = 1$

$f(3) = 4$

$g(3) = 4$



$$f(x) = \frac{3x+2}{x-1} \quad \text{funz omografiche}$$

$$f(x) = \frac{ax+b}{cx+d} \quad \text{con } a, b, c, d \in \mathbb{R}$$

$$\begin{cases} y = f(x) \\ y = g(x) \end{cases}$$ per calcolare le intersezioni

$$\frac{4}{4-x} = (x-1)^2$$

$$4 = (4-x)(x-1)^2$$

$$4 = (4-x)(x^2-2x+1)$$

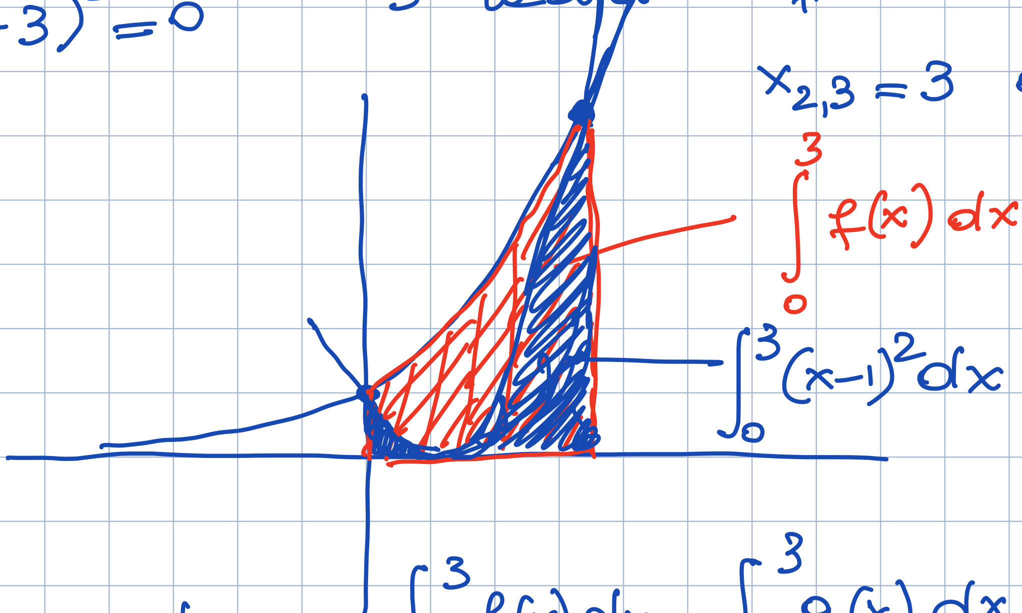
$$\cancel{4} = 4x^2 - x^3 - 8x + 2x^2 + \cancel{4} - x$$

$x^3 - 6x^2 + 9x = 0$

$x(x^2 - 6x + 9) = 0$

$x(x-3)^2 = 0$ 3 radici $x_1 = 0$ semplice

$x_{2,3} = 3$ doppia



l'area cercata $= \int_0^3 f(x)\,dx - \int_0^3 g(x)\,dx =$

$$= -\underbrace{\int_0^3 \frac{4\cdot(-1)}{4-x}\,dx}_{I_1} - \underbrace{\int_0^3 (x-1)^2\,dx}_{I_2} =$$

In I_1 $\quad y = 4 - x \quad dy = -dx$

se $x = 0 \Rightarrow y = 4$

se $x = 3 \Rightarrow y = 1$

$$I_1 = -\int_4^1 \frac{4\,dy}{y} = -4\left[\log|y|\right]_4^1 = -4(\log 1 - \log 4)$$

$$= +4\log 4$$

$$I_2 = \left[\frac{1}{3}(x-1)^3\right]_0^3 = \frac{1}{3}8 - \frac{1}{3}(-1) = 3$$

$$A = I_1 - I_2 = 4\log 4 - 3.$$

• Sia $f(x) = \dfrac{x^2 - 3x + 3}{x^3 - 2x^2 + x}$? primitiva $F(x)$: $F(2) = \log 8$

$$\int \frac{x^2 - 3x + 3}{x^3 - 2x^2 + x}\, dx$$

fratto $n < m$

n ↑ grado num

m ↑ grado den.

$$x^3 - 2x^2 + x = x\left(x^2 - 2x + 1\right) = x(x-1)^2$$

$$\frac{x^2 - 3x + 3}{x^3 - 2x^2 + x} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = \frac{A(x-1)^2 + B \cdot x(x-1) + Cx}{x(x-1)^2}$$

$$\left[\frac{\quad}{(x^2+1)^2} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+1)^2} \right]$$

$$= \frac{Ax^2 + A - 2Ax + Bx^2 - Bx + Cx}{x(x-1)^2} = \frac{(A+B)x^2 + x(-2A-B+C) + A}{x(x-1)^2}$$

$$\forall x: \quad x^2 = (A+B)x^2 \iff \begin{cases} 1 = A+B \\ -3 = -2A - B + C \\ 3 = A \end{cases}$$

$$-3x = (-2A-B+C)x$$

$$3 = A$$

sistema lineare di 3 eqz in 3 incognite

$$\begin{cases} A = 3 \\ B = -2 \\ C = -3 + 2A + B = -3 + 6 - 2 = 1 \end{cases}$$

$$\frac{x^2-3x+3}{x^3-2x^2+x} = \frac{3}{x} - \frac{2}{x-1} + \frac{1}{(x-1)^2}$$

$$F(x) = \int \frac{3}{x}\,dx - 2\int \frac{1}{x-1}\,dx + \int \frac{1}{(x-1)^2}\,dx$$

$$= 3\log|x| - 2\log|x-1| - \frac{1}{x-1} + c$$

$$F(2) = \cancel{3\log 2} - \cancel{2\log 1} - 1 + c = \underbrace{\log 8}_{}$$

$$\log 2^3$$

$$\cancel{3\log 2}$$

$$\Rightarrow c = 1$$

$$F(x) = 3\log|x| - 2\log|x-1| - \frac{1}{x-1} + 1$$

- ? Media integrale di $f(x) = \frac{1}{\sin x}$ su $\left[\frac{\pi}{3}, \frac{\pi}{2}\right]$

? $$\int_{\frac{\pi}{3}}^{\pi/2} \frac{1}{\sin x}\,dx \qquad = \frac{1}{\frac{\pi}{2}-\frac{\pi}{3}} \cdot \int_{\frac{\pi}{3}}^{\pi/2} \frac{1}{\sin x}\,dx$$

cerco una primitiva

$$\sin x = \frac{2\,\mathrm{tg}\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}} \qquad \cos x = \frac{1-\mathrm{tg}^2\frac{x}{2}}{1+\mathrm{tg}^2\frac{x}{2}}$$

$$\int \frac{1}{\sin x}\,dx = \int \frac{\overbrace{1+tg^2\frac{x}{2}\,dx}^{dt}}{2\,\underbrace{tg\frac{x}{2}}_{t}} = \int \frac{1}{t}\,dt = (*)$$

$$t = tg\frac{x}{2} \qquad dt = \frac{1}{2}\left(1+tg^2\frac{x}{2}\right)dx$$

$$D(tg\,x) = D\left(\frac{\sin x}{\cos x}\right) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \begin{cases} \dfrac{1}{\cos^2 x} \\ 1 + tg^2 x \end{cases}$$

$$D\left(tg\frac{x}{2}\right) = \left(1+tg^2\frac{x}{2}\right)\cdot\frac{1}{2}$$

$$(*) = \log|t| = \log\left|tg\frac{x}{2}\right| = \text{primitive}$$

$$\text{media} = \frac{1}{\frac{\pi}{2}-\frac{\pi}{3}}\cdot\left[\log\left|tg\frac{x}{2}\right|\right]_{\frac{\pi}{3}}^{\pi/2} =$$

$$= \frac{6}{\pi}\left(\underbrace{\log\left|\underbrace{tg\frac{\pi}{4}}_{1}\right|}_{0} - \log\left|tg\frac{\pi}{6}\right|\right) = \frac{6}{\pi}\left(-\log\frac{1}{\sqrt{3}}\right)$$

$$= \frac{6}{\pi}\left(-\log(\sqrt{3})^{-1}\right)$$

$$= \frac{6}{\pi}\cdot\log\sqrt{3}$$

$$tg\frac{\pi}{6} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$