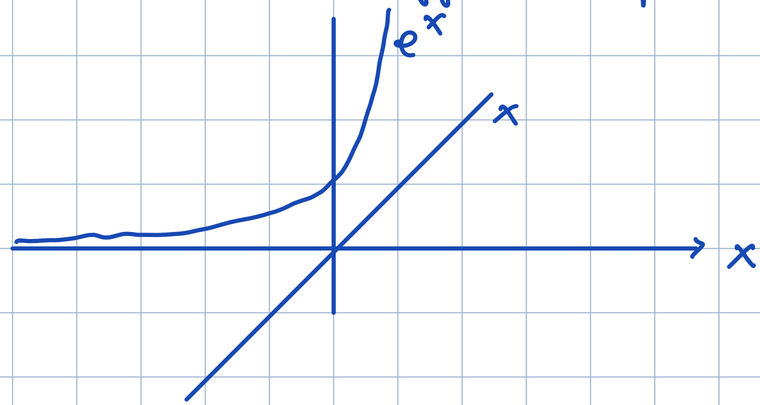
13/11/24

- $$\lim_{x \to +\infty} \frac{e^x}{x} = \frac{+\infty}{+\infty} \overset{(H)}{=} \lim_{x \to +\infty} \frac{e^x}{1} = +\infty$$

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} \overset{(H)}{=} \lim_{x \to x_0} \frac{f'(x)}{g'(x)}$$
sotto opportune ip.

e^x ha ordine di ∞ maggiore rispetto a x per $x \to +\infty$



- $$\lim_{x \to +\infty} \frac{e^x}{x^{10}} \overset{(H)}{=} \lim_{x \to +\infty} \frac{e^x}{10 \cdot x^9} \overset{(H)}{=} \dots\dots \overset{(H)}{=} \lim_{x \to +\infty} \frac{e^x}{10!} = +\infty$$

- $$\lim_{x \to +\infty} \frac{e^x}{x^\alpha} = +\infty \quad \forall \alpha > 0$$

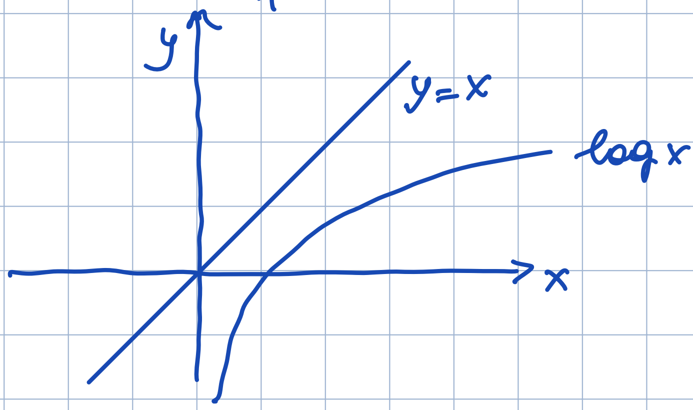
- $$\lim_{x \to +\infty} \frac{e^{\beta x}}{x^\alpha} = +\infty \quad \forall \alpha > 0, \ \forall \beta > 0$$

- $$\lim_{x\to+\infty} \frac{\overset{\to+\infty}{\log x}}{\underset{\to+\infty}{x}} = \frac{+\infty}{+\infty}$$

$$\overset{(H)}{=} \lim_{x\to+\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$$

- $$\lim_{x\to+\infty} \frac{(\log x)^{\beta}}{x^{\alpha}} = 0$$

$\forall \alpha > 0, \ \forall \beta > 0$

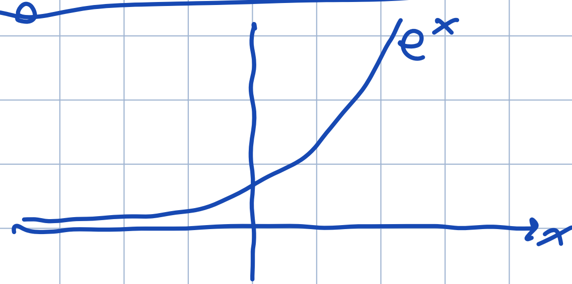


$$\lim_{x\to+\infty} \frac{\sqrt[3]{\log x}}{x^5} = 0 \qquad \beta = \frac{1}{3} \qquad \alpha = 5$$

$$\sqrt[3]{\log x} = (\log x)^{1/3}$$

$$\log^2 x = (\log x)^2 \neq \log x^2$$

- $$\lim_{x\to-\infty} \underset{\downarrow\, -\infty}{x}\, \overset{\to 0}{e^{x}} = \infty \cdot 0$$



$$= \lim_{x\to-\infty} \frac{\overset{\to -\infty}{x}}{\underset{\to+\infty}{e^{-x}}} \overset{(H)}{=} \lim_{x\to-\infty} \frac{1}{-e^{-x}} = \frac{1}{\infty} = 0$$

• Studio $f(x) = \log x$ $\quad$ $\mathrm{dom}(f) = (0, +\infty)$

$f'(x) = \frac{1}{x}$ $\quad$ $\mathrm{dom}(f') \subseteq \mathrm{dom}(f)$

$\quad = (0, +\infty)$

$f(x) = 2\sin x + \cos(2x)$

Dopo aver studiato dom e eventuali sim.
verificare che f ha periodo $T = 2\pi$ e studiare
f su $[0, 2\pi]$.

• $\mathrm{dom}(f) = \mathbb{R}$

• sim. $f(-x) = 2\sin(-x) + \cos(-2x) = -2\sin x + \cos(2x)$

$$\neq \begin{cases} f(x) \\ -f(x) \end{cases}$$ $\Rightarrow$ f non ha simmetrie rispetto all'origine.

• verifico il periodo. f è periodica di periodo T

se $f(x+T) = f(x)$ $\quad \forall x \in \mathrm{dom}(f)$

$\cos(2x + 4\pi)$

$f(x+2\pi) = 2\sin(x+2\pi) + \cos(2(x+2\pi))$

$= 2\sin x + \cos(2x)$ $\quad$ per la periodicità di $\sin x$ e $\cos x$

$= f(x)$

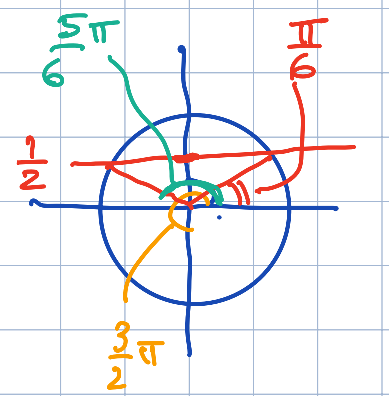
Studio f su $[0, 2\pi]$ $\quad$ $f(x) = 2\sin x + \cos(2x)$

non cerco asintoti perché f è def su $[0, 2\pi]$
(non può andare a $\pm\infty$).

- $f'(x) = 2\cos x - \sin(2x)\cdot 2$ $\qquad$ $\mathrm{dom}(f') \subseteq \mathrm{dom}(f)$

$= 2\cos x - 4\sin x \cdot \cos x$ $\qquad$ $\sin(2x) = 2\sin x \cdot \cos x$

$= \underbrace{2\cos x}_{F_1} \cdot \underbrace{(1 - 2\sin x)}_{F_2}$



- ? pti stazionari $\quad f'(x) = 0$

$\cos x = 0 \iff x = \frac{\pi}{2}$ o $x = \frac{3}{2}\pi$

$(1 - 2\sin x) = 0 \iff \sin x = \frac{1}{2}$

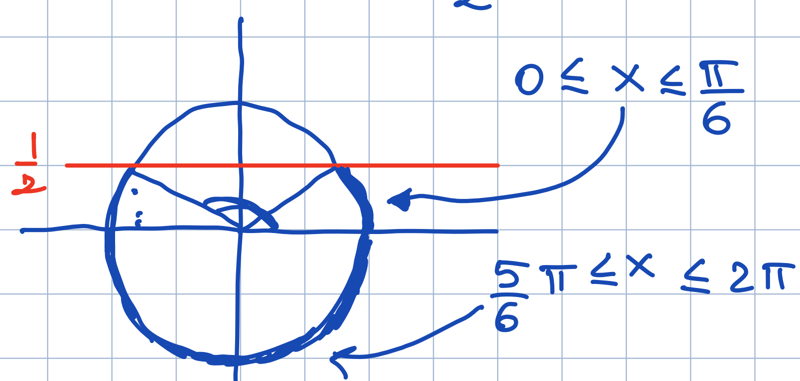
$x = \frac{\pi}{6}$ o $x = \frac{5}{6}\pi$

4 punti stazionari

- $f'(x) \geq 0$

$F_1(x) = 2\cos x \geq 0 \iff 0 \leq x \leq \frac{\pi}{2}$ o $\frac{3}{2}\pi \leq x \leq 2\pi$

$F_2(x) = 1 - 2\sin x \geq 0 \iff \sin x \leq \frac{1}{2}$



	0		$\pi/6$		$\pi/2$		$5/6\,\pi$		$\frac{3}{2}\pi$		2π
F_1		$+$		$+$	0	$-$		$-$	0	$+$	
F_2		$+$	0	$-$		$-$	0	$+$		$+$	
f'		$+$	0	$-$	0	$+$	0	$-$	0	$+$	
f		$\nearrow$	$-$	$\searrow$	$-$	$\nearrow$	$-$	$\searrow$	$-$	$\nearrow$	

$\frac{\pi}{6}$ è pto di max rel

pto di min rel

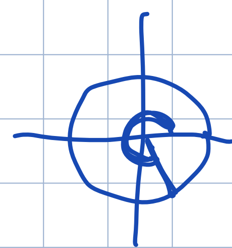
pto di max rel

pto di min rel

$f(x) = 2\sin x + \cos(2x)$

$$f(0) = 2\sin 0 + \cos 0 = 1 = f(2\pi)$$

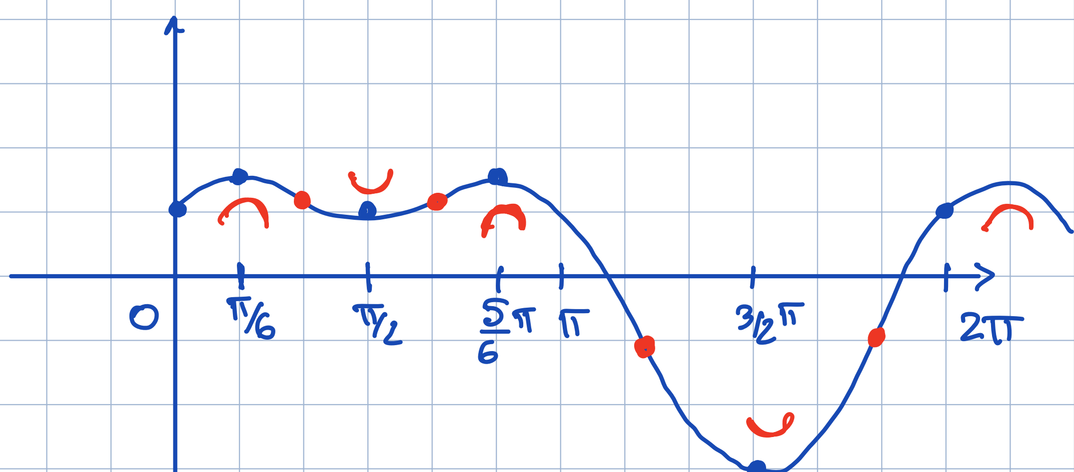
$$f\left(\frac{\pi}{6}\right) = 2\cdot\frac{1}{2} + \frac{1}{2} = \frac{3}{2}$$



$$f\left(\frac{\pi}{2}\right) = 2 + \cos(\pi) = 1$$

$$f\left(\frac{5\pi}{6}\right) = 2\cdot\frac{1}{2} + \underbrace{\cos\left(\frac{5}{3}\pi\right)}_{\frac{1}{2}} = \frac{3}{2}$$

$$f\left(\frac{3}{2}\pi\right) = 2\cdot(-1) + \cos(3\pi) = -3$$



f è cresc in $\left(0, \frac{\pi}{6}\right) \cup \left(\frac{\pi}{2}, \frac{5}{6}\pi\right) \cup \left(\frac{3}{2}\pi, 2\pi\right)$

e decresc in $\left(\frac{\pi}{6}, \frac{\pi}{2}\right) \cup \left(\frac{5}{6}\pi, \frac{3}{2}\pi\right)$

$x = \frac{\pi}{6}, \frac{5}{6}\pi$ sono pti di max ass.

$x = \frac{3}{2}\pi$ è pto di min ass.

$x = \frac{\pi}{2}$ è pto di min rel.

Ci sono almeno 4 pti di flesso in $[0, 2\pi]$, sono deducibili analizzando la concav e convess nei pti di estremo relativo.

$$f(x) = 7\sin\left(\frac{\pi}{2}(|x| - x)\right) + 2\log\left(x(|x| + x) + 1\right)$$

$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

$$f(x) = \begin{cases} \cancel{7\sin\left(\frac{\pi}{2}(x - x)\right)} + 2\log\Big(\underbrace{x(x + x) + 1}_{(2x^2+1)}\Big) & \text{se } x \geq 0 \\ 7\sin\left(\frac{\pi}{2}(-x - x)\right) + \underbrace{\cancel{2\log\Big(x \cdot \underbrace{(-x + x)}_{0} + 1\Big)}}_{\log 1 = 0} & \text{se } x < 0 \end{cases}$$

$$= \begin{cases} 2\log(2x^2 + 1) & \text{se } x \geq 0 \\ -7\sin(\pi x) & \text{se } x < 0 \end{cases}$$

$$f(x) = e^{|\log x| - \frac{1}{3x}} = \exp\left(|\log x| - \frac{1}{3x}\right)$$

(tema del 1/07/2016)

$$\operatorname{dom}(f) = \begin{cases} x > 0 & \text{log} \\ x \neq 0 & \text{frazione} \end{cases} \Rightarrow \operatorname{dom}(f) = (0, +\infty)$$

no simmetrie perché il dom non è sim.

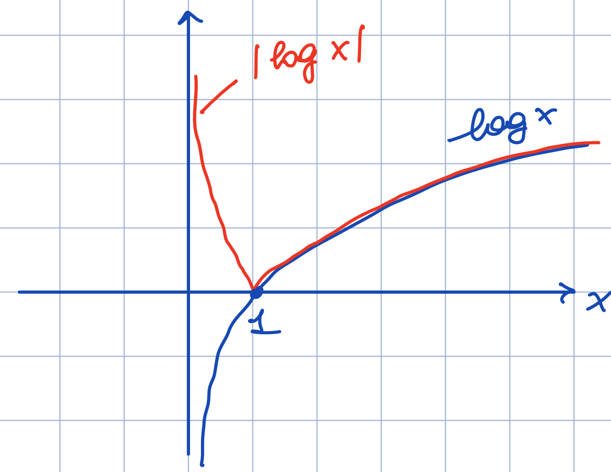
- $$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} e^{|\log x| - \frac{1}{3x}} =$$

$$= \lim_{x\to 0^+} e^{-\log x - \frac{1}{3x}} =$$

$$= \lim_{x\to 0^+} \frac{1}{e^{\log x + \frac{1}{3x}}} =$$

$$= \lim_{x\to 0^+} \frac{1}{\underbrace{e^{\log x}}_{x} \cdot e^{1/3x}} =$$

$$= \lim_{x\to 0^+} \frac{1}{\underbrace{\underbrace{x}_{\to 0^+}\,\underbrace{e^{1/3x}}_{\to +\infty}}_{\to +\infty}} = 0$$



$$|\log x| = \begin{cases} \log x & \text{se } x \geq 1 \\ -\log x & \text{se } 0 < x < 1 \end{cases}$$

$$\lim_{x\to 0^+} x e^{1/(3x)} = \lim_{t\to +\infty} \frac{1}{3t} e^t = +\infty$$

$$t = \frac{1}{3x} \Rightarrow x = \frac{1}{3t}$$

se $x \to 0^+ \Rightarrow t \to +\infty$

$$\Rightarrow \lim_{x\to 0^+} f(x) = 0$$

non c'è as. verticale in $x=0$

$$\lim_{x\to+\infty} e^{|\log x| - \frac{1}{3x}} = \lim_{x\to+\infty} e^{\underbrace{\log x}_{+\infty} - \underbrace{\frac{1}{3x}}_{0}} = +\infty$$

$\nexists$ as. orizz per $x \to +\infty$, ci potrebbe essere as. obl.

$$m = \lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{e^{\log x - \frac{1}{3x}}}{x} = \lim_{x\to+\infty} \frac{\cancel{e^{\log x}} \cdot e^{-\frac{1}{3x}}}{\cancel{x}} = 1$$

$x = e^{\log x}$ $\qquad \frac{1}{3x} \to 0$

$m = 1$

$$q = \lim_{x\to+\infty} (f(x) - x) = \lim_{x\to+\infty} \left(e^{\log x - \frac{1}{3x}} - x\right) =$$

$$= \lim_{x\to+\infty} \left(x \cdot e^{-1/3x} - x\right) = \lim_{x\to+\infty} \underbrace{x}_{+\infty} \underbrace{\left(e^{-1/3x} - 1\right)}_{0} =$$

$1/3x \to 0$

$$= \lim_{t\to0^+} \frac{1}{3t}\left(e^{-t} - 1\right) =$$

$t = \frac{1}{3x}$ $\qquad x = \frac{1}{3t}$ $\qquad \lim_{t\to0} \frac{e^t - 1}{t} = 1$

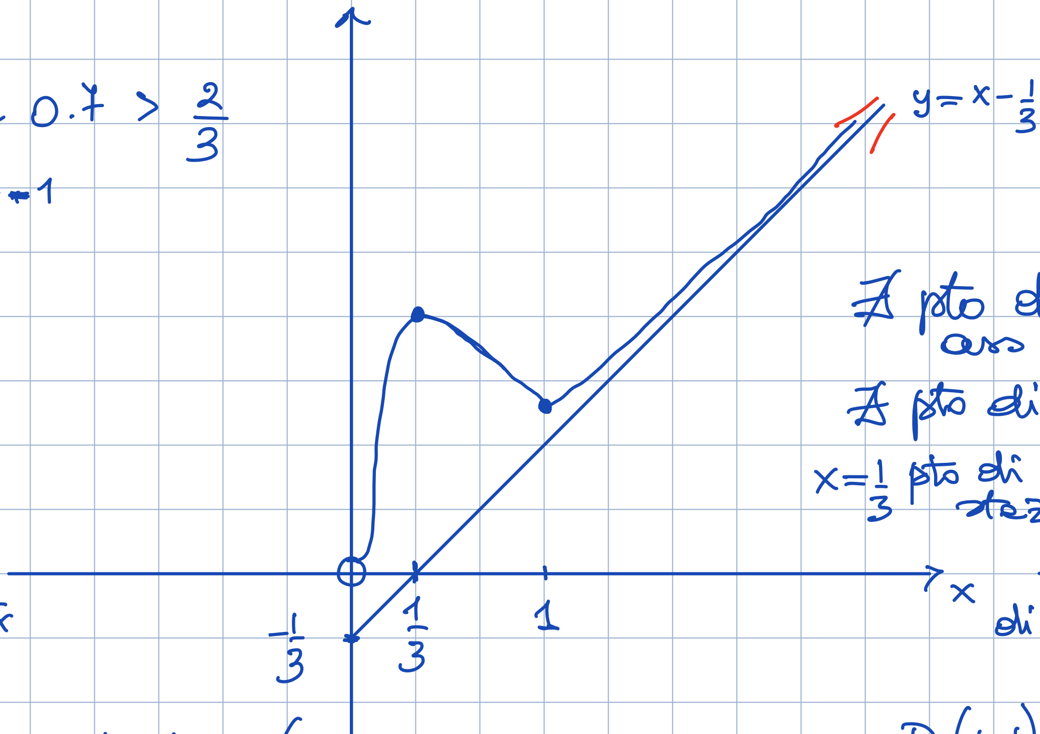
$y = -t$ se $t \to 0^+ \Rightarrow y \to 0^-$

$$= \lim_{y\to0^-} \frac{1}{-3y}\left(e^y - 1\right) = -\frac{1}{3} \lim_{y\to0^-} \frac{e^y - 1}{y} = -\frac{1}{3} = q$$

as. obl $y = x - \frac{1}{3}$

$f(1) = e^{-1/3} \sim 0.7 > \frac{2}{3}$

$f\left(\frac{1}{3}\right) = e^{-\log\frac{1}{3} - 1}$



∄ pto di min ass.

∄ pto di max ass

$x = \frac{1}{3}$ pto di max rel staz.

$x = 1$ = pto di min rel. angoloso

$f(x) = e^{|\log x| - \frac{1}{3x}}$

$D(|x|) = \frac{|x|}{x}$

$D\left(\frac{1}{x}\right) = -\frac{1}{x^2}$

$$f'(x) = e^{|\log x| - \frac{1}{3x}} \cdot \left(\frac{|\log x|}{\log x} \cdot \frac{1}{x} - \frac{1}{3}\left(-\frac{1}{x^2}\right) \right)$$

$$= e^{|\log x| - \frac{1}{3x}} \cdot \left(\frac{|\log x|}{\log x} \cdot \frac{1}{x} + \frac{1}{3x^2} \right)$$

$$= \begin{cases} e^{\log x - \frac{1}{3x}} \cdot \left(\frac{1}{x} + \frac{1}{3x^2} \right) & x > 1 \\ e^{-\log x - \frac{1}{3x}} \cdot \left(-\frac{1}{x} + \frac{1}{3x^2} \right) & 0 < x < 1 \end{cases}$$

$\cancel{x \geq 1}$

$$|\log x| = \begin{cases} \log x & \text{se } x \geq 1 \\ -\log x & \text{se } 0 < x < 1 \end{cases}$$

? $\mathrm{dom}(f') = \{ x \in \mathrm{dom}(f) : \log x \neq 0 \}$

$= (0, 1) \cup (1, +\infty)$

? che tipo di non derivabilità ho in $x = 1$

? $f'_-(1)$ e $f'_+(1)$

se $\exists \lim_{x \to x_0^\mp} f'(x) = \ell_{1,2} \Rightarrow f'_\mp(x_0) = \ell_{1,2}$

$$f'_-(1) = \lim_{x \to 1^-} f'(x) = \lim_{x \to 1^-} e^{-\log x - \frac{1}{3x}} \left(-\frac{1}{x} + \frac{1}{3x^2} \right) = e^{-\frac{1}{3}} \cdot \left(-1 + \frac{1}{3} \right)$$

$$= -\frac{2}{3} e^{-1/3} < 0$$

trovo che $\lim_{x \to 1^-} f'(x) \exists \Rightarrow$ coincide con $f'_-(1)$

$$f'_+(1) \overset{?}{=} \lim_{x \to 1^+} f'(x) = \lim_{x \to 1^+} e^{\log x - \frac{1}{3x}} \left(\frac{1}{x} + \frac{1}{3x^2} \right) = e^{-1/3} \left(1 + \frac{1}{3} \right)$$

$$= \frac{4}{3} e^{-1/3} > 0$$

$\Rightarrow$ $f'_-(1) \neq f'_+(1)$ entrambi finiti

$x = 1$ è punto angoloso

$f'_- < 0$ $f'_+ > 0$

deduco che 1 è pto di min relativo.

$\boxed{0 < x < 1}$ studio $f'(x) = 0$ e poi $f'(x) \geq 0$

$$f'(x) = \underbrace{e^{-\log x - \frac{1}{3x}}}_{\neq 0} \left(-\frac{1}{x} + \frac{1}{3x^2} \right) = 0 \iff -3x + 1 = 0$$

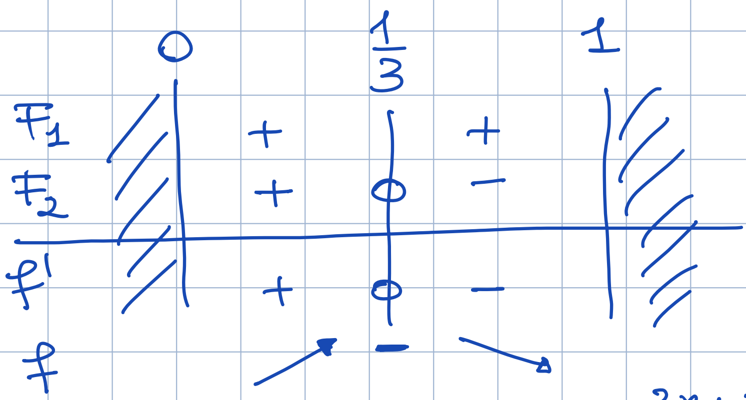
$$\left(\frac{-3x + 1}{3x^2} \right) \iff x = \frac{1}{3}$$

è acc

$f'(x)=0 \qquad x=\frac{1}{3}$ è pto staz.

$f'(x)\geq 0 \qquad F_1(x) = e^{\cdots\cdots} > 0 \quad \forall x \in \text{dom} f$

$F_2(x) = -\frac{3x+1}{3x^2} \geq 0 \iff \qquad x \leq \frac{1}{3}$



$\boxed{x>1}$

$$f'(x) = \underbrace{e^{\log x - \frac{1}{3x}}}_{\neq 0} \Big(\overbrace{\frac{1}{x} + \frac{1}{3x^2}}^{\frac{3x+1}{3x^2}} \Big)$$

$f'(x)=0 \iff \frac{3x+1}{3x^2}=0 \iff 3x+1=0 \iff x=-\frac{1}{3}$

non è acc

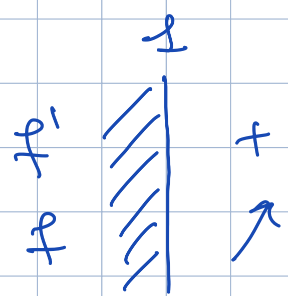
per $x>1$ non ci sono pti staz.

$f'(x)\geq 0 \qquad F_1(x)=e^{\cdots\cdots}>0 \quad \forall x\in\text{dom} f$

$F_2(x) = \frac{3x+1}{3x^2} > 0 \quad \forall x \in \text{dom}(f)$

se $x>1 \Rightarrow 3x+1>0, \; 3x^2>0$

$f'(x)>0 \quad \forall x>1$



	0		$1/3$		1	
f'	‖	$+$	|	$-$	|	$+$
f	‖	↗		↘	↓	↗