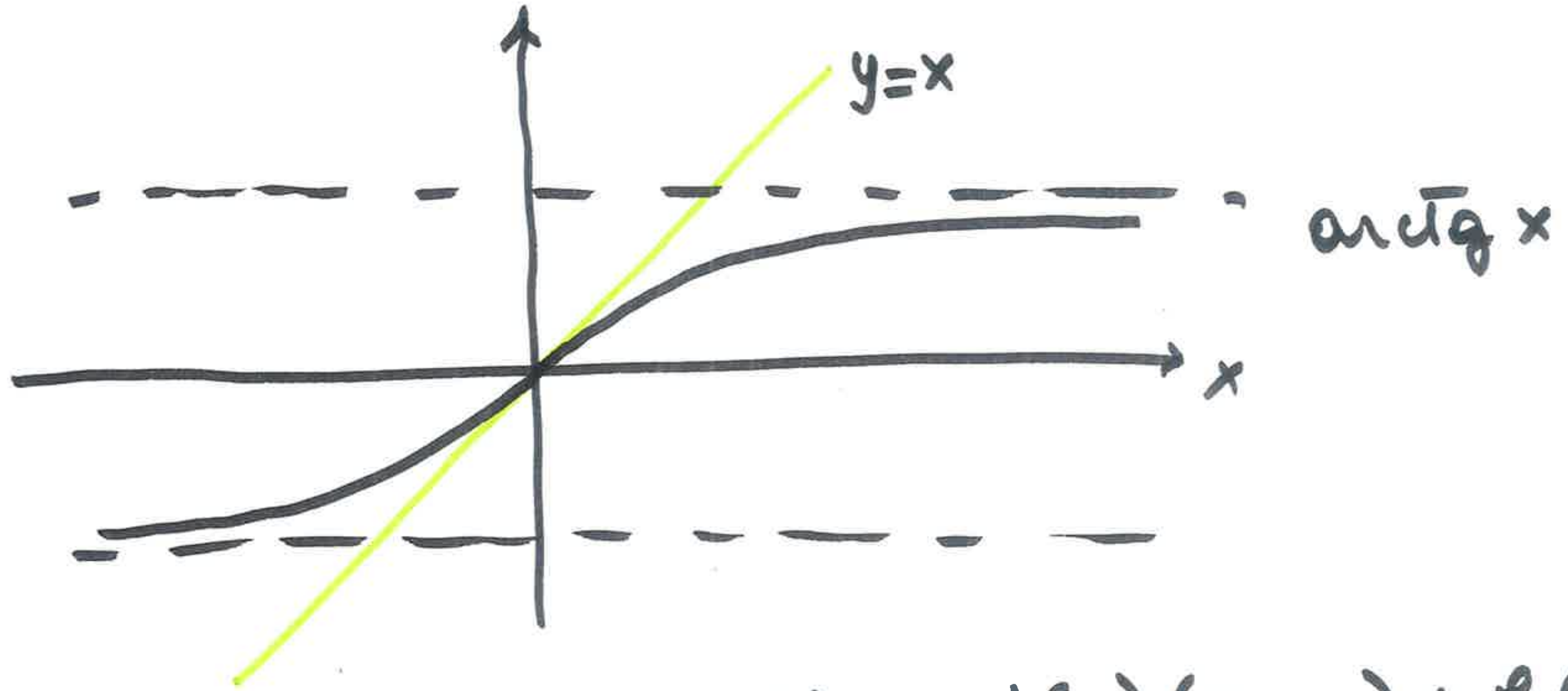


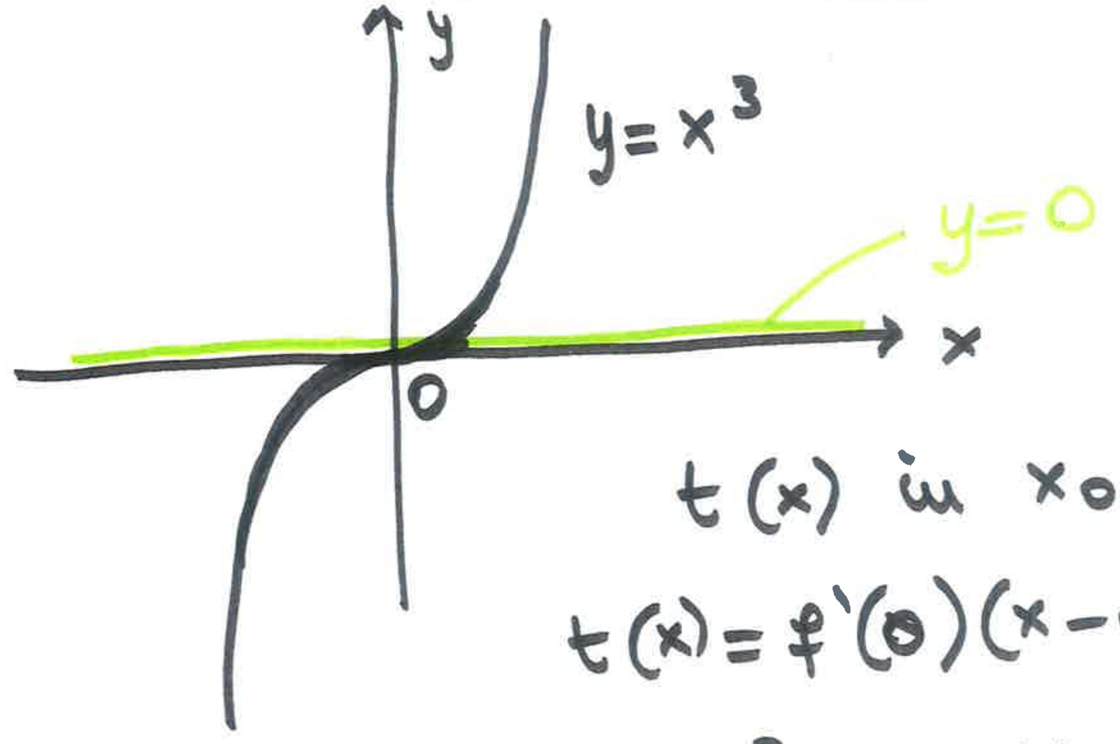
$t(x)$ in $x_0 = 0$

$t(x) = f'(0)(x-0) + f(0)$

$\left(t(x) = f'(x_0)(x - x_0) + f(x_0) \right)$

$f'(x) = \dfrac{1}{1+x^2}$ $\quad f'(0) = 1$

$t(x) = 1 \cdot x + 0 = x$



$t(x)$ in $x_0 = 0$

$t(x) = f'(0)(x-0) + f(0)$

$f(x) = x^3 \qquad f'(x) = 3x^2$

$f'(0) = 0$

$\Rightarrow t(x) = 0(x-0) + 0 = 0$

$f(x) = \sqrt[3]{x} = x^{1/3}$? $t(x) = f'(x_0)(x - x_0) + f(x_0)$

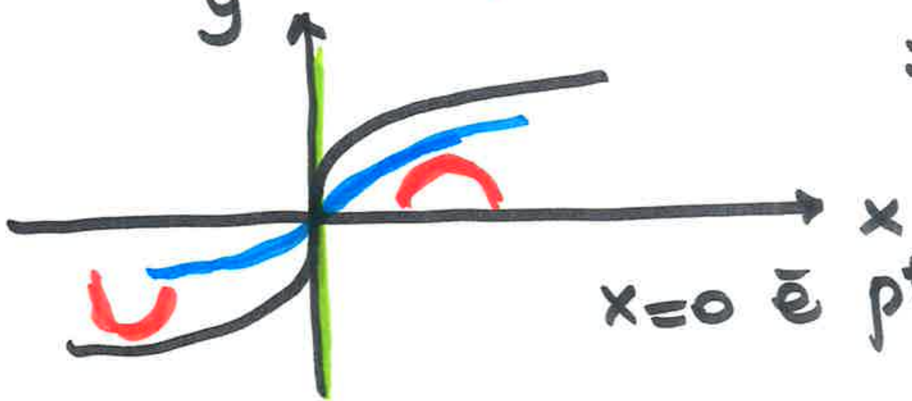
$x_0 = 0$

$f'(x) = \frac{1}{3} x^{\frac{1}{3}-1} = \frac{1}{3\sqrt[3]{x^2}}$ $f'(x)$ non ē definita per $x = 0$

$x = 0 \notin dom(f')$ ē un pto di non deriv.

$f'_{\mp}(0) = \lim_{x \to 0^{\mp}} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^{\mp}} \frac{\sqrt[3]{x} - 0}{x - 0} =$ $x^{1/3}$

$= \lim_{x \to 0^{\mp}} \frac{1}{x^{2/3}} = \frac{1}{0^+} = +\infty$



$f'_-(0) = f'_+(0) = +\infty$

pto a tg vert

$x = 0$ ē pto di flesso a tg verticale

$f(x) = e^{1/x}$.

- dom $(f) = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, +\infty)$
- simmetrie $f(-x) = e^{-1/x}$ $\neq f(x) = e^{1/x}$; $\neq -f(x) = -e^{1/x}$ $\Rightarrow$ f non ha sim.

- limiti

$\lim_{x \to -\infty} e^{\overset{\to 0^-}{1/x}} = e^0 = 1$ y = 1 as oriz sx

$\lim_{x \to +\infty} e^{\overset{\to 0^+}{1/x}} = e^0 = 1$ y = 1 as oriz dx

$\Rightarrow$ y = 1 as oriz completo

$\lim_{x \to 0^-} \underbrace{e^{\overset{\to -\infty}{1/x}}}_{\to 0} = 0$

$\lim_{x \to 0^+} \underbrace{e^{\overset{\to +\infty}{1/x}}}_{\to +\infty} = +\infty$ as vert dx

- discont ? $f(x) = e^{1/x}$

$x=0 \notin dom(f) \Rightarrow$ non può essere pto di disc

$\forall x \neq 0$ f ē comp di f continue $\Rightarrow$ ē cont.

- $f'(x) = e^{1/x} \cdot \left(-\frac{1}{x^2}\right)$ $\frac{1}{x} = x^{-1}$, $D(x^{-1}) = -1 \cdot x^{-2} = -\frac{1}{x^2}$

$dom(f') = dom(f) \Rightarrow \nexists$ pti di non deriv.

- ? pti stazionari $f'(x)=0 \iff e^{1/x} = 0$ mai

$\nexists$ pti staz.

- cresc/decresc. $f'(x) \geq 0$ $-\frac{e^{1/x}}{x^2} \geq 0$ mai

$e^{1/x} > 0$ (exp ē sempre >0)

$x^2 > 0$ $\forall x \neq 0$

$\Rightarrow f'(x) \leq 0$ $\forall x \in dom(f)$

	0	
f'	$-$	$-$
f	$\searrow$	$\searrow$

f e descresc în
$(-\infty, 0) \cup (0, +\infty)$

$$f'(x) = -\frac{e^{1/x}}{x^2}$$

$$\left[\left(\frac{N}{D}\right)' = \frac{N' \cdot D - N \cdot D'}{D^2}\right]$$

$$-\; f''(x) = \oplus \frac{\oplus \frac{e^{1/x}}{\cancel{x^2}} \cdot \cancel{x^2} \oplus e^{1/x} \cdot 2x}{x^4} = \frac{e^{1/x} + e^{1/x} \cdot 2x}{x^4} =$$

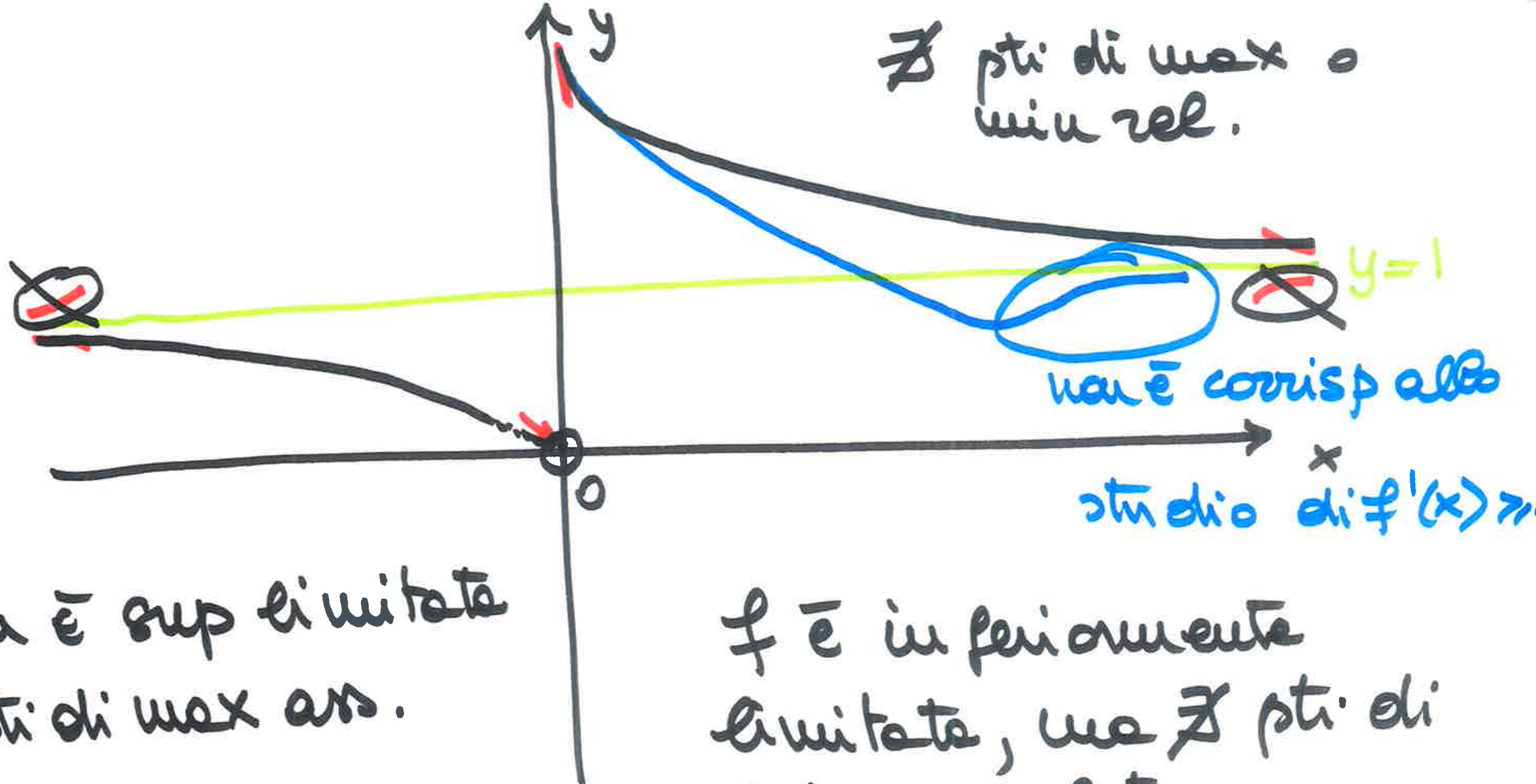
$$= \frac{e^{1/x}(1 + 2x)}{x^4}$$

$F_1(x) = e^{1/x} \geq 0 \quad \forall x \in dom f$

$F_2(x) = 1 + 2x \geq 0 \quad x \geq -\frac{1}{2}$

$F_3(x) = x^4 > 0 \quad \forall x \in dom f$

$? x \in dom(f) \quad f''(x) \geq 0$



∄ pti di max o min rel.

f non è sup limitata
∄ pti di max ass.

f è inferiormente limitata, ma ∄ pti di min assoluto

Adesso f non è definita in 0, quindi non posso calcolare $f'_-(0)$, ma voglio capire come f si avvicina a 0 da sx.

Calcolo $\boxed{\lim_{x\to 0^-} f'(x)} = \lim_{x\to 0^-} -\frac{e^{1/x}}{x^2} = \frac{0}{0}$ (*)

$$\lim_{x\to x_0} \frac{f(x)}{g(x)} \overset{(H)}{=} \lim_{x\to x_0} \frac{f'(x)}{g'(x)}$$

$e^{1/x} = t$

$\frac{1}{x} = \log t$

se $x \to 0^-$

$t \to 0^+$

$\frac{1}{x^2} = \log^2 t$

$\lim_{t\to 0^+} t^\alpha \cdot \log t = 0 \quad \forall \alpha > 0$

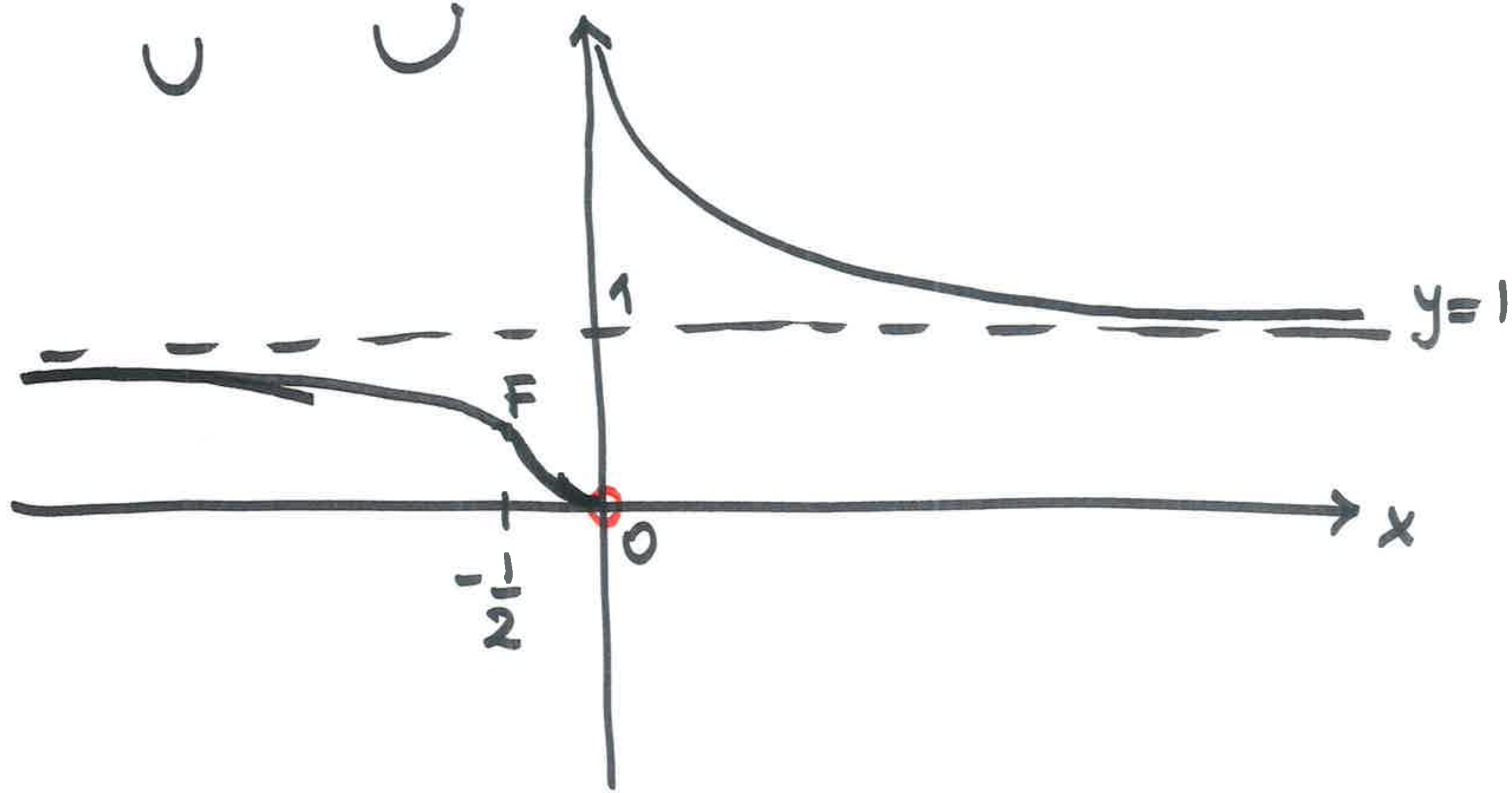
(*) $= \lim_{t\to 0^+} -(t \cdot \log^2 t) =$

$= -\lim_{t\to 0^+} \underbrace{(t^{1/2} \cdot \log t)}_{\to 0} \cdot \underbrace{(t^{1/2}\log t)}_{\to 0} = 0$

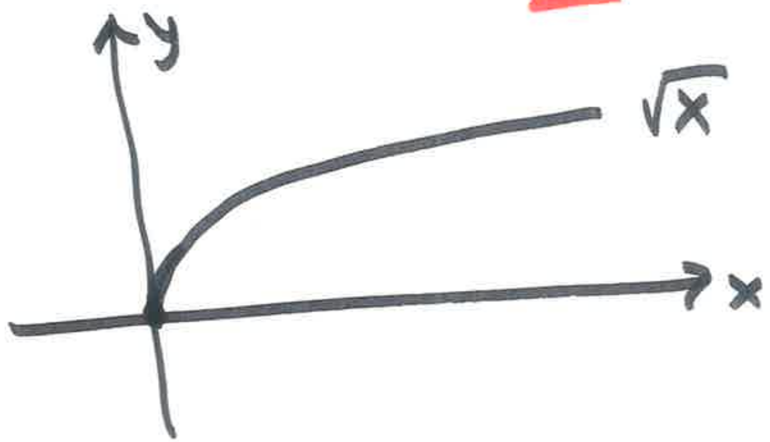
	$-\frac{1}{2}$		0	
F_1	$+$		$+$	$+$
F_2	$-$	0	$+$	$+$
F_3	$+$		$+$	$+$
f''	$-$		$+$	$+$
f	$\cap$		$\cup$	$\cup$

~~Ricordandosi del tutto fatto vedendo~~ -

$$\lim_{x \to 0^-} f'(x) = 0$$



$f(x) = \sqrt{x}$ su $[0, +\infty)$ $\qquad f \in C^0([0, +\infty))$



$$f'(x) = \frac{1}{2\sqrt{x}}$$

$\mathrm{dom}(f') = (0, +\infty)$

f' è continua su $(0, +\infty)$

$\Rightarrow f \notin C^1([0, +\infty))$

$f \in C^1((0, +\infty))$

$C^k(I)$ spazi di funzioni

f ē $C^0(\mathbb{R})$

f ē derivabile

ma $f \notin C^1(\mathbb{R})$ perchē

$x=0$ ē pto di disc per f'

$$\lim_{x\to 0} \frac{\sin(x^2)}{x} = \lim_{x\to 0} \underbrace{\frac{\sin(x^2)}{x^2}}_{\to 1} \cdot x = 0$$

$t = x^2$

$x \to 0$

$t \to 0$

$$\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$$

$f(x) = o(g(x))$ per $x \to 0$

$\boxed{\sin(x^2) = o(x)}$ per $x \to 0$