Esercitazione 14/10/2024

1) variante tema 20/06/24

$$f(x) = \frac{\operatorname{arctg}\left(\frac{x}{\sqrt{2+|x|}}\right)\cdot\left(\log^2 x - \log x\right)^{\sqrt{5}}}{\sqrt{5x-1}\cdot\sqrt[5]{x-1}}$$

? dom (f) , ? $x \in$ dom (f) : $f(x) < 0$

$\boxed{\text{dom } f}$

- $\operatorname{arctg}\left(\frac{x}{\sqrt{2+|x|}}\right)$

arctg non dà restrizioni

devo chiedere però $\boxed{\sqrt{2+|x|} \neq 0}$

- $\underbrace{\left(\log^2 x - \log x\right)}_{t}^{\sqrt{5}}$

$y = t^\alpha$ con α irrazionale

$\alpha = \sqrt{5} > 0$ $\quad t^\alpha = t^{\sqrt{5}}$ è definita se $t \geq 0$

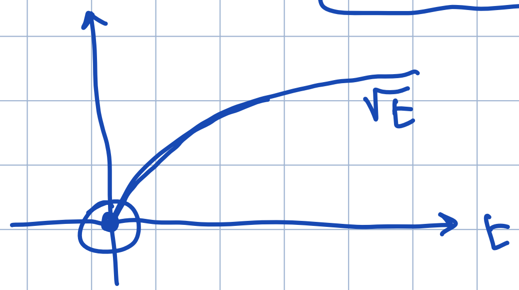
devo chiedere $\boxed{\left(\log^2 x - \log x\right) \geq 0}$

devo chiedere $\boxed{x > 0}$ per dare senso a $\log x$

• $\sqrt{5x-1}$: $\boxed{\begin{array}{l} 5x-1 \geq 0 \\ \sqrt{5x-1} \neq 0 \end{array}}$ per dare senso a $\sqrt{\ }$

perché è a denomin.

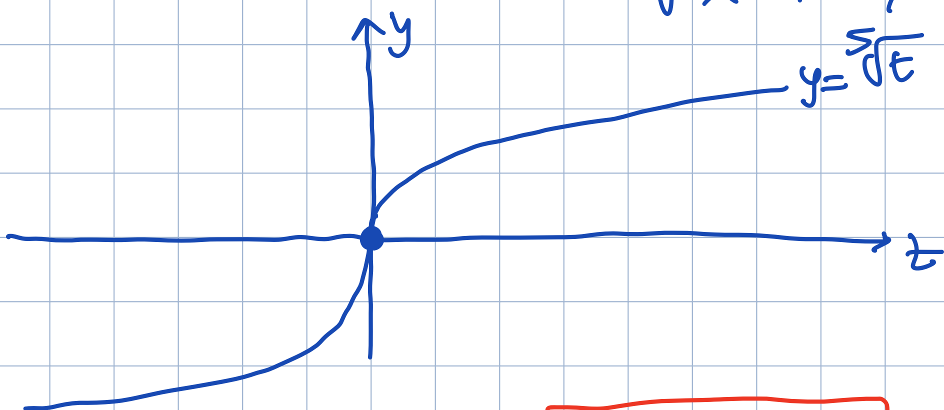
$y = \sqrt{t}$



$\sqrt{5x-1} \neq 0 \equiv 5x-1 \neq 0$

$\boxed{5x-1 > 0}$

• $\sqrt[5]{x-1}$ radice di indice dispari ha dom $= \mathbb{R}$
ma è a denominatore
devo chiedere $\sqrt[5]{x-1} \neq 0$

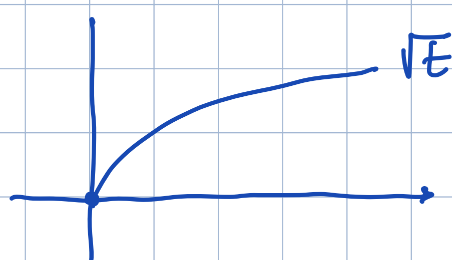


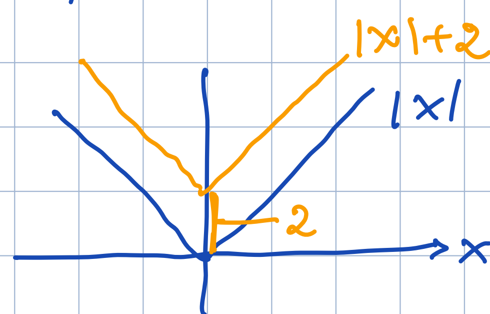
$\sqrt[5]{t}$

$\sqrt[5]{x-1} \neq 0 \iff \boxed{x-1 \neq 0}$

$$\begin{cases} \sqrt{2+|x|} \neq 0 \\ \log^2 x - \log x \geq 0 \\ x > 0 \\ 5x-1 > 0 \\ x-1 \neq 0 \end{cases} \qquad \begin{cases} 2+|x| \neq 0 \quad \forall x \in \mathbb{R} \\ 0 < x \leq 1 \ \text{ o } \ x \geq e \\ x > 0 \\ x > 1/5 \\ x \neq 1 \end{cases}$$

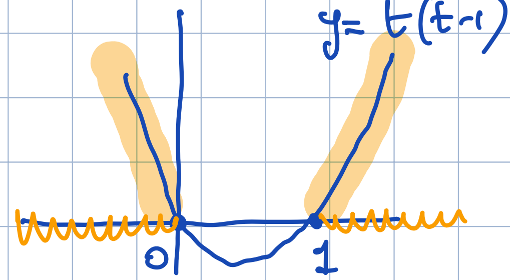
$y = \sqrt{t}$





- $\log^2 x - \log x \geq 0$ $\qquad t = \log x$

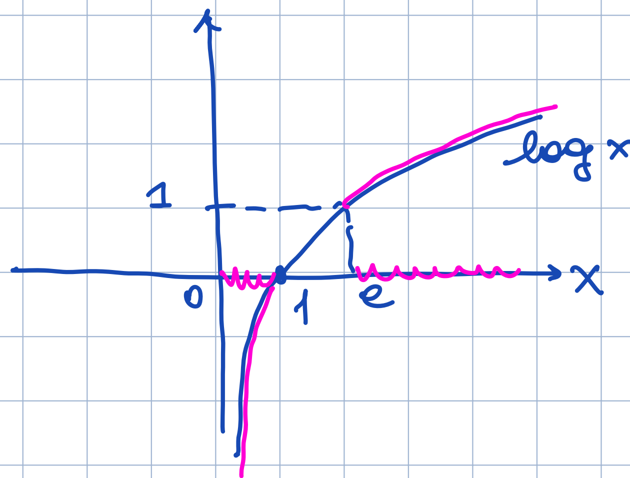
$t^2 - t \geq 0$ $\qquad \underbrace{t(t-1)}_{y} \geq 0$



$t \leq 0$ o $t \geq 1$

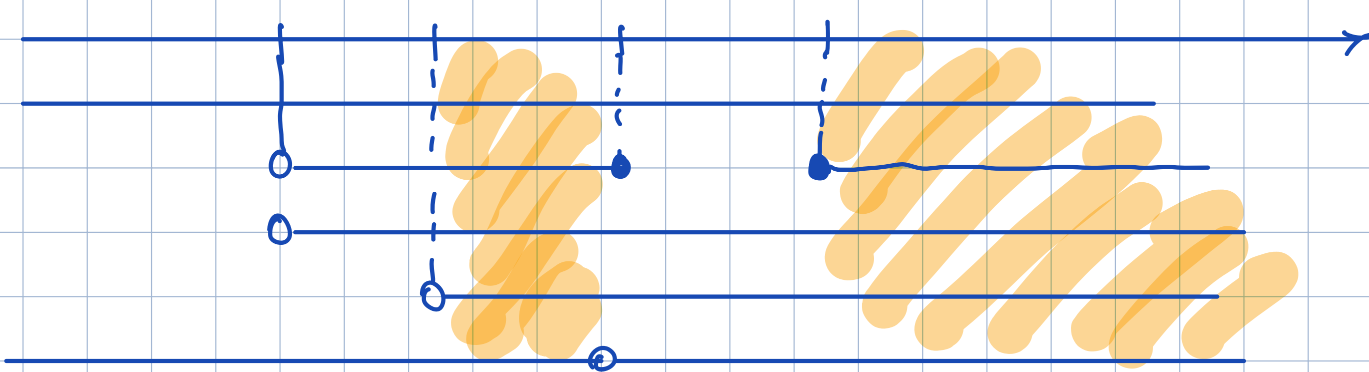
$\log x \leq 0$ o $\log x \geq 1$

$0 < x \leq 1$ o $x \geq e$



$$\begin{cases} \cancel{2 + |x| \neq 0} \quad \forall x \in \mathbb{R} \\ 0 < x \leq 1 \text{ o } x \geq e \\ x > 0 \\ x > 1/5 \\ x \neq 1 \end{cases}$$

{ vuole dire ∩ uso le linee cont. non + e −



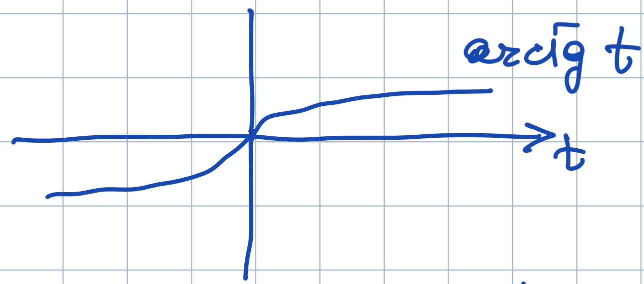
$\frac{1}{5} < x < 1 \quad \cup \quad x \geq e$

$\text{dom}(f) = \left(\frac{1}{5}, 1\right) \cup [e, +\infty)$

? $A = \{ x \in \text{dom}(f) : f(x) < 0 \}$

$$f(x) = \frac{\operatorname{arctg}\left(\frac{x}{\sqrt{2+|x|}}\right) \cdot \left(\log^2 x - \log x\right)^{\sqrt{5}}}{\sqrt{5x-1} \cdot \sqrt[5]{x-1}}$$

- $F_1(x) = \operatorname{arctg}\left(\frac{x}{\sqrt{2+|x|}}\right)$



? $x : F_1(x) \geq 0$

$\Updownarrow$

$$\frac{x}{\sqrt{2+|x|}} \geq 0$$

$\operatorname{arctg}(t) \geq 0 \iff t \geq 0$

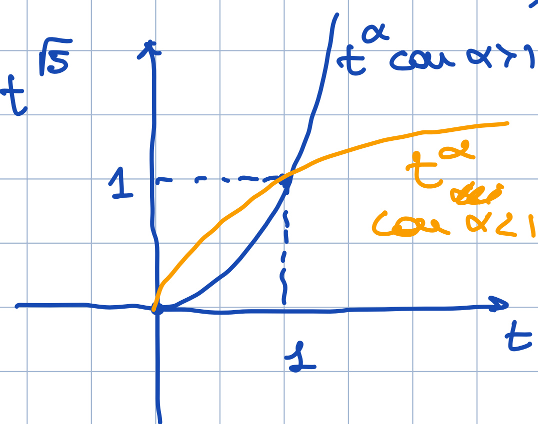
$N = x \geq 0$

$D = \sqrt{2+|x|} > 0$ $\quad \forall x \in \mathbb{R}$ a patto che non si annulli (ma non si annulla)

$F_1(x) \geq 0$ sse $\boxed{x \geq 0}$

- $F_2(x) = \left(\log^2 x - \log x\right)^{\sqrt{5}}$

$y = t^{\sqrt{5}}$



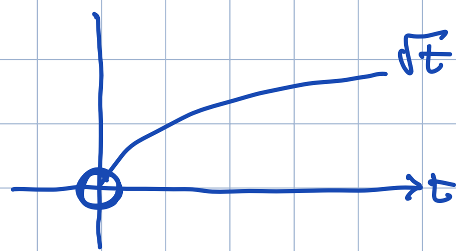
$F_2(x) \geq 0$ sse

$$\log^2 x - \log x \geq 0$$

$\Updownarrow$

$$\boxed{0 < x \leq 1 \quad \text{o} \quad x \geq e}$$

- $F_3(x) = \sqrt{5x-1} > 0$



sse $\quad 5x-1 \neq 0$

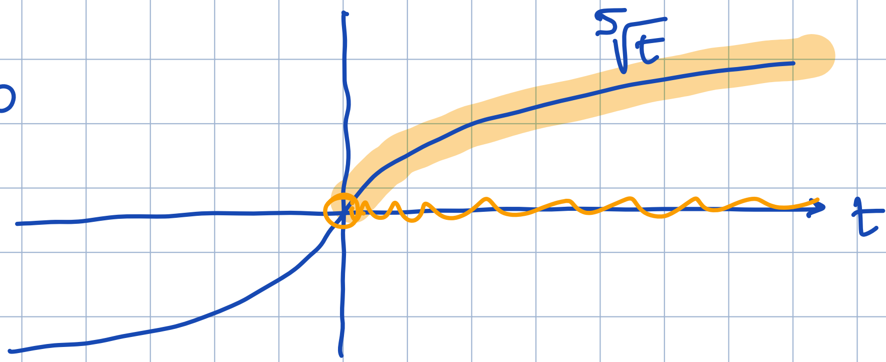
$$\boxed{x \neq \frac{1}{5}}$$

• $F_4(x) = \sqrt[5]{x-1} > 0$

$\Updownarrow$

$x - 1 > 0$

$x > 1$



		0		$1/5$		1		e	
F_1	$-$	0	$+$		$+$		$+$		$+$
F_2			$+$		$+$	0		0	$+$
F_3	$+$		$+$	$\nexists$	$+$		$+$		$+$
F_4	$-$		$-$		$-$	$\nexists$	$+$		$+$
				$\nexists$	$-$	$\nexists$		0	$+$

$f(x) < 0$

$\frac{1}{5} < x < 1$

$A = \left(\frac{1}{5}, 1\right)$

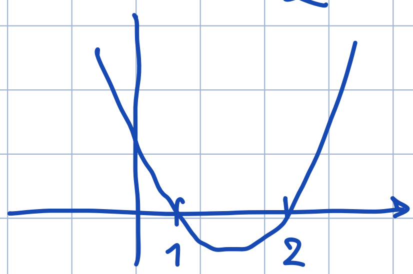
variante 12/01/24

$$f(x) = \left(e^{1/(x^2-3x+2)} - e\right) \cdot \operatorname{arctg}(7x - x^2)$$

? dom f , ? $x \in \text{dom}(f) : f(x) \geq 0$

$\boxed{\text{dom } f}$ $\quad x^2 - 3x + 2 \neq 0$ perché denominatore

$$x_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \begin{cases} \frac{3+1}{2} = 2 \\ \frac{3-1}{2} = 1 \end{cases}$$



$x \neq 1$ e $x \neq 2$

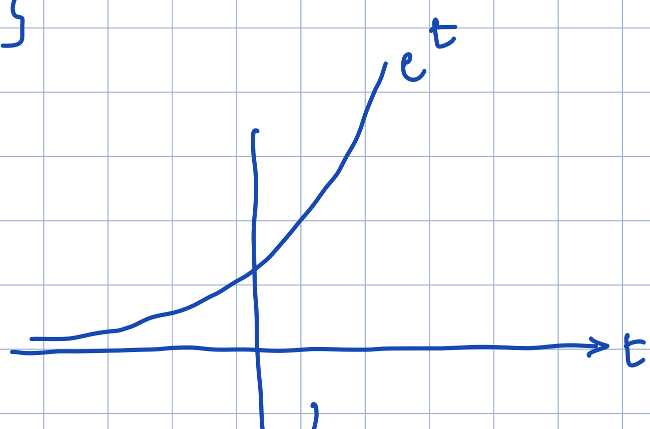
$$\text{dom}(f) = \mathbb{R} \setminus \{1, 2\}$$

? $A = \{x \in \text{dom}(f) : f(x) \geq 0\}$

- $F_1(x) = e^{\frac{1}{x^2-3x+2}} - e \geq 0$

$$e^{\frac{1}{x^2-3x+2}} \geq e^1$$

$$\log\left(e^{\frac{1}{x^2-3x+2}}\right) \overset{?}{\geq} \log e$$



$$\frac{1}{x^2-3x+2} \geq 1$$

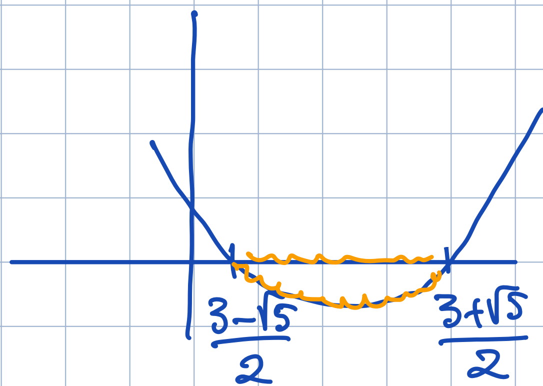
$$\frac{1}{x^2-3x+2} - 1 \geq 0$$

$$\frac{1-x^2+3x-2}{x^2-3x+2} \geq 0$$

$$\frac{-x^2+3x-1}{x^2-3x+2} \geq 0$$

$N \geq 0$ $\qquad -x^2+3x-1 \geq 0$

$$x^2-3x+1 \leq 0 \qquad x_{1,2} = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$



$$\frac{3-\sqrt{5}}{2} \leq x \leq \frac{3+\sqrt{5}}{2}$$

$D > 0$ $\qquad x^2-3x+2>0 \qquad x_1 = 2 \quad x_2 = 1$

$$x < 1 \quad o \quad x > 2$$

discuto il segno di F_1

		$\frac{3-\sqrt5}{2}$		1		2		$\frac{3+\sqrt5}{2}$	
$N \geq 0$	−	0	+		+		+	0	−
$D > 0$	+		+		−		+		+
	−	0	+		−		+	0	−

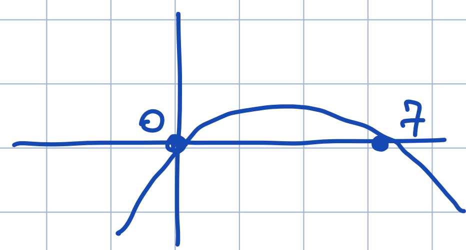
$F_1(x) \geq 0: \quad \frac{3-\sqrt{5}}{2} \leq x < 1 \quad$ or $\quad 2 < x \leq \frac{3+\sqrt{5}}{2}$

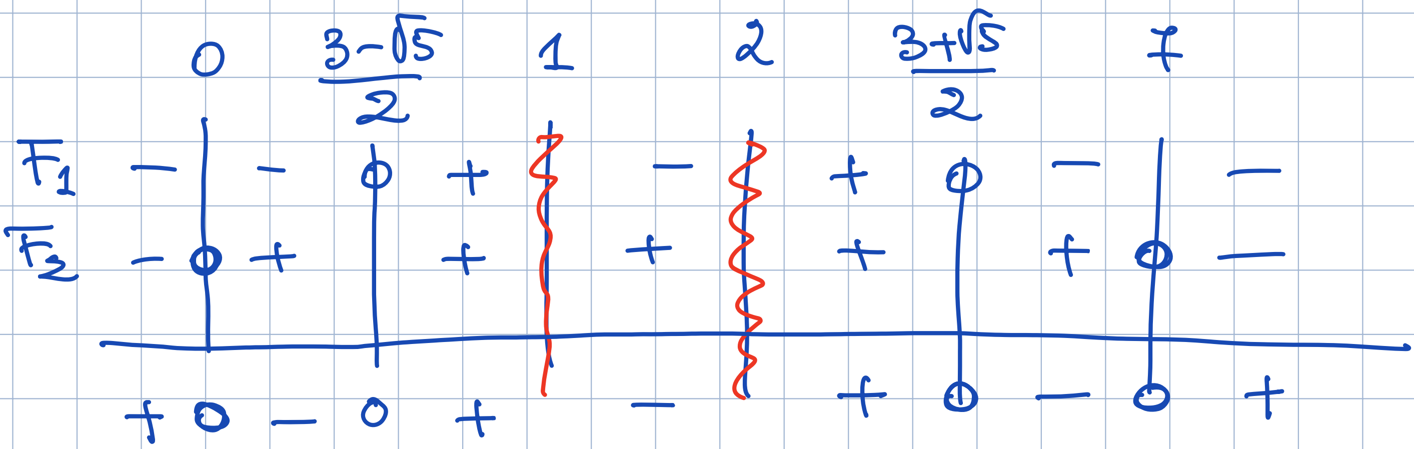
$x(7-x)$

- $F_2(x) \qquad \operatorname{arctg}\left(7x - x^2\right) \geq 0 \iff 7x - x^2 \geq 0$

$x_1 = 0 \qquad x_2 = 7$

$\iff 0 \leq x \leq 7$





$$A = (-\infty, 0] \cup \left[\frac{3-\sqrt{5}}{2}, 1\right) \cup \left(2, \frac{3+\sqrt{5}}{2}\right] \cup [7, +\infty)$$

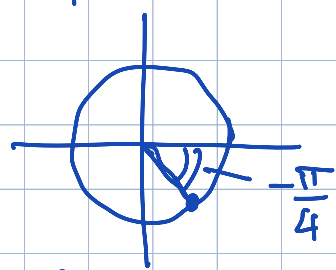
variante del 16/04/2019

? radici complesse cubiche di $w = \dfrac{2^{-|1-i\sqrt{19}|^2}\cdot(7+7i)\cdot(1-i)^{40}}{\underbrace{2\cdot i^{32}}_{2}+e^{3i\pi}}$

? $z\in\mathbb{C} : z^3 = w = \rho e^{i\theta}$

- $|1-i\sqrt{19}|^2 = \left(\sqrt{1+19}\right)^2 = 20$
- $(7+7i) = 7(1+i) = 7\,\frac{2}{\sqrt{2}}\Big(\underbrace{\frac{\sqrt{2}}{2}}_{\cos\frac{\pi}{4}} + \underbrace{\frac{\sqrt{2}}{2}}_{\sin\frac{\pi}{4}}\,i\Big) = 7\cdot\sqrt{2}\,e^{i\frac{\pi}{4}}$
- $(1-i) = \frac{2}{\sqrt{2}}\left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\,i\right)$

$= \sqrt{2}\cdot e^{-i\pi/4}$

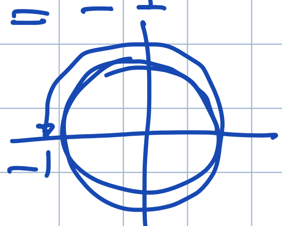


$(1-i)^{40} = \Big(\underbrace{\sqrt{2}}_{2^{1/2}}\cdot e^{-i\pi/4}\Big)^{40} = 2^{\cancel{\frac{40}{2}}\,20}\cdot e^{-i\frac{\pi}{\cancel{4}}\cdot\cancel{40}\,10} = 2^{20}\cdot e^{-i10\pi}$

$e^{-i10\pi} = 1$

$(1-i)^{40} = 2^{20}$

- $2\cdot i^{32} = 2\cdot\left(e^{i\frac{\pi}{2}}\right)^{32} = 2\cdot\underbrace{e^{i16\pi}}_{1} = 2$
- $e^{3i\pi} = -1$



$$w = \frac{\cancel{2^{-20}} \cdot 7\sqrt{2} \cdot e^{i\frac{\pi}{4}} \cdot \cancel{2^{20}}}{\underbrace{2 + (-1)}_{1}} = 7\sqrt{2}\, e^{i\frac{\pi}{4}}$$

$$\rho = 7\sqrt{2} \qquad \vartheta = \frac{\pi}{4}$$

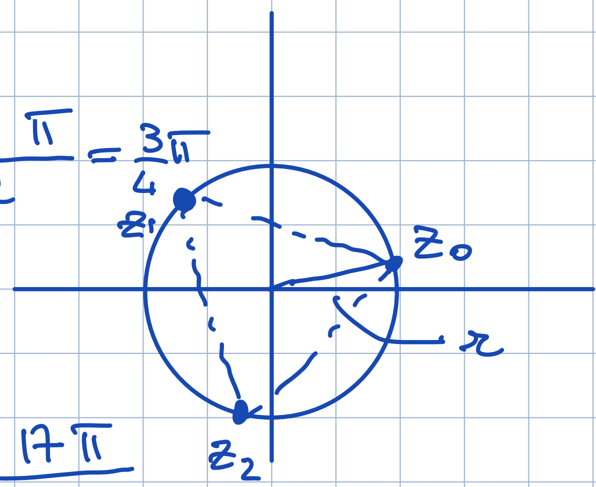
$$z^3 = 7\sqrt{2} \cdot e^{i\pi/4} \qquad n = 3$$

$$r = \sqrt[n]{\rho} = \sqrt[3]{7\sqrt{2}}$$

$$\varphi_0 = \frac{\vartheta}{n} = \frac{\pi/4}{3} = \frac{\pi}{12} \qquad z_0 = \sqrt[3]{7\sqrt{2}} \cdot e^{i\frac{\pi}{12}}$$

$$\varphi_1 = \varphi_0 + \frac{2\pi}{n} = \frac{\pi}{12} + \frac{2\pi}{3} = \frac{9\pi}{12} = \frac{3\pi}{4}$$

$$z_1 = \sqrt[3]{7\sqrt{2}}\, e^{i\frac{3}{4}\pi}$$



$$\varphi_2 = \varphi_1 + \frac{2\pi}{n} = \frac{3}{4}\pi + \frac{2\pi}{3} = \frac{17\pi}{12}$$

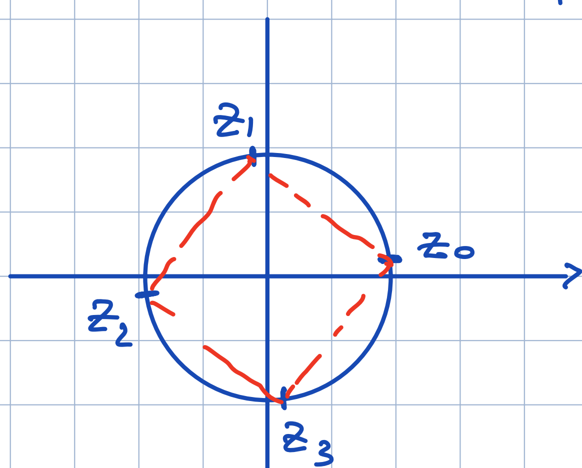
$$z_2 = \sqrt[3]{7\sqrt{2}}\, e^{i\frac{17}{12}\pi}$$

? $$z^4 = 7\sqrt{2}\, e^{i\pi/4} \qquad \rho = 7\sqrt{2} \qquad \vartheta = \frac{\pi}{4}$$

$$r = \sqrt[4]{7\sqrt{2}}$$

$$\varphi_0 = \frac{\vartheta}{n} = \frac{\pi}{16}$$

$$z_0 = \sqrt[4]{7\sqrt{2}} \cdot e^{i\pi/16}$$



Tema 02/07/2019

? $A = \left\{ z \in \mathbb{C} : \begin{cases} (\mathrm{Re}\, z)^2 - \frac{3}{2}(z + \bar{z}) + 2 \leq 0 \\ \mathrm{Im}\left(\frac{1}{z}\right) \leq 0 \\ \mathrm{Re}\left(\frac{z^2}{i \cdot \mathrm{Re}\, z}\right) \leq 3 \end{cases} \right\}$

e calcolarne l'area.

- $(\mathrm{Re}\, z)^2 - \frac{3}{2}(z + \bar{z}) + 2 \leq 0$ $\qquad z = x + iy$

$$x^2 - \frac{3}{2}(x + \cancel{iy} + x - \cancel{iy}) + 2 \leq 0$$

$$x^2 - 3x + 2 \leq 0 \qquad x_{1,2} = \frac{3 \pm \sqrt{9-8}}{2} = \begin{cases} 2 \\ 1 \end{cases}$$

$$\boxed{1 \leq x \leq 2}$$

- $\mathrm{Im}\left(\frac{1}{z}\right) \leq 0$ $\qquad z \neq 0 \qquad z = x + iy$

$$\mathrm{Im}\left(\frac{1}{x+iy}\right) \leq 0$$

$$\frac{1}{x+iy} = \frac{1}{x+iy} \cdot \frac{x-iy}{x-iy} = \frac{x-iy}{x^2+y^2} =$$

$$= \frac{x}{x^2+y^2} - i \frac{y}{x^2+y^2}$$

$$\mathrm{Im}\left(\frac{1}{x+iy}\right) = -\frac{y}{x^2+y^2} \qquad x^2+y^2 \neq 0 \iff \begin{cases} x \neq 0 \\ y \neq 0 \end{cases}$$

$\operatorname{Im}\left(\frac{1}{x+iy}\right) \le 0 \iff \frac{-y}{x^2+y^2} \le 0 \iff \frac{y}{x^2+y^2} \geq 0$

$\iff y \geq 0$ perché $x^2+y^2>0$ (ho chiesto $z \neq 0$)

- $\operatorname{Re}\left(\frac{z^2}{i \cdot \operatorname{Re} z}\right) \le 3$

$\operatorname{Re} z \neq 0 \iff x \neq 0$

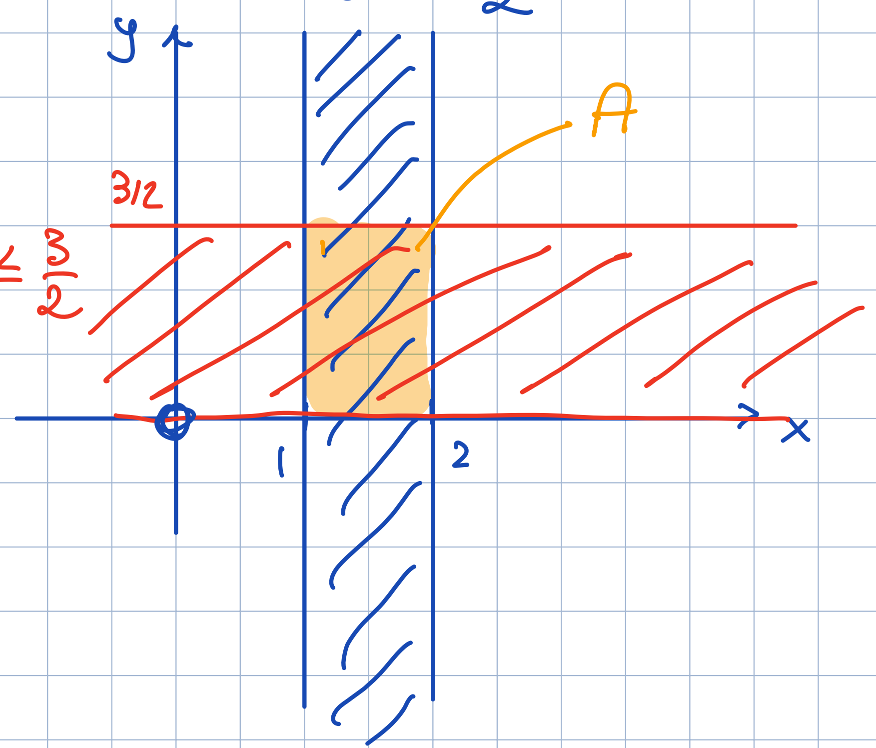
$z = x+iy$

$\operatorname{Re}\left(\frac{x^2-y^2+2xyi}{ix}\right) = \operatorname{Re}\left(\frac{(x^2-y^2+2xyi)(-ix)}{(ix)(-ix)}\right)$

$= \operatorname{Re}\left(\frac{-x(x^2-y^2)i + 2x^2y}{x^2}\right)$

$= \frac{2\cancel{x^2}y}{\cancel{x^2}} \le 3$ cioè $y \le \frac{3}{2}$

$\begin{cases} 1 \le x \le 2 \\ y \geq 0 \\ y \le \frac{3}{2} \end{cases}$ $0 \le y \le \frac{3}{2}$



Area $= 1 \cdot \frac{3}{2}$

Variante del 03/02/20

? $A = \{ z \in \mathbb{C} : (z^2 + 4i) \cdot \mathrm{Re}\left(2z(\bar{z} + 2i) - 2i(z - \bar{z})\right.$

$\left. - \underbrace{|z-2|^2} - 2(z + \bar{z})\right) = 0 \}$

$|z - 2|^2 = |x + iy - 2|^2 = |(x-2) + iy|^2$

$x + iy$

$= (x-2)^2 + y^2$