The Lotka-Volterra prey-predator system

(Lions and gazelles, bacteria and antibodies, ...)

The ode system

$$\begin{cases} \dfrac{\mathrm{d}y_1}{\mathrm{d}t} = C_1 y_1 \left(1 - b_1 y_1 - d_2 y_2\right), \quad t > t_0 \\ \dfrac{\mathrm{d}y_2}{\mathrm{d}t} = -C_2 y_2 \left(1 - b_2 y_2 - d_1 y_1\right), \quad t > t_0 \\ y_1(t_0) = y_{10} \\ y_2(t_0) = y_{20} \end{cases} \tag{1}$$

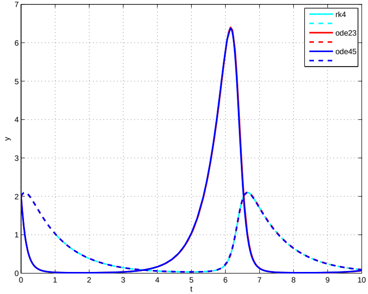
describes the evolution of two populations ($y_1(t)$ and $y_2(t)$) that coexist and are in competition in the time interval ($t \in [t_0, T]$). $y_1(t_0)$ and $y_2(t_0)$ are the inital number of individuals of each population, while C_1 and C_2 represent the growth rates of the two populations. The coefficients d_1 and d_2 govern the type of interaction between the two populations, while b_1 and b_2 are related to the available quantity of nutrients.

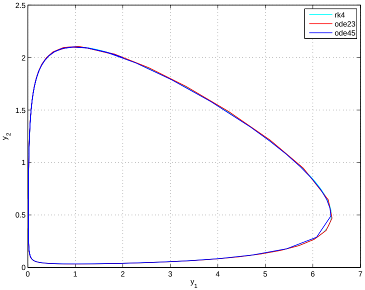
Example Let us set $C_1 = 2,\ C_2 = 1,\ d_1 = 1,\ d_2 = 2,\ b_1 = b_2 = 0,$ $y_{10} = 2,\ y_{20} = 2,\ t_0 = 0,\ T = 10.$
We aim at simulating the evolution of the two populations.
Solution Write a script file in order to:

1. define the problem data,
2. solve the ode system by RK4 and the functions `ode23` and `ode45` of matlab
3. plot the solution vs the time
4. plot the solution in the phase space.

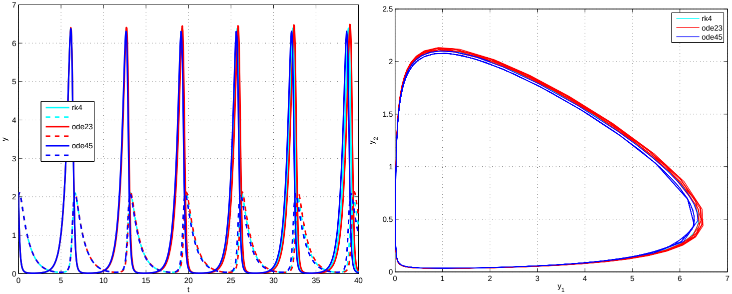
For RK4 put $h = 0.01$, while both `ode23` and `ode45` do not require the time-step since they are adaptive formulas (they dinamically compute the time step). The calling instruction for both `ode23` and `ode45` are:
`[tn,un]=odexx(@fun,tspan,y0,options);` if `fun` is defined in a m-file,
`[tn,un]=odexx(fun,tspan,y0,options);` if `fun` is defined by a function-handle.

Solutions for $t \in [0, 10]$





Fix now $T = 40$.



Notice that all the solutions are not in phase, the less accurate are those computed by adaptive methods.

It is necessary to change the tolerance for the choice of the time step:

```
options=odeset('RelTol',1.e-6);
[tn,un]=ode23(@flotka,tspan,y0,options);
```

The new results are all comparable