f sia non derivabile nei punti di raccordo, quindi per cautela si esclude sempre il punto di raccordo, poi si verifica cosa succede in quel punto.

Osservo che per $x>0$ la f' è definita per $x\neq e$

($x=e$ annulla i denominatori)

Quindi dovrò studiare f' in $x=0$ e in $x=e$

$\boxed{x=0}$

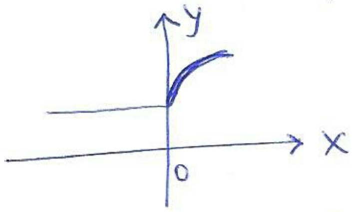
$$f'_-(0)=\lim_{x\to 0^-}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^-}\frac{1-1}{x}=0$$

$$f'_+(0)=\lim_{x\to 0^+}\frac{f(x)-f(0)}{x-0}=\lim_{x\to 0^+}\frac{\cancel{1}+\sqrt{x|\log x-1|}-\cancel{1}}{x}=$$

$$=\lim_{x\to 0^+}\frac{\sqrt{|\log x-1|}}{\sqrt{x}}=\frac{+\infty}{0}=+\infty$$

non ci sono forme indeterminate!

$$f'_-(0)=0\ ,\quad f'_+(0)=+\infty \Rightarrow x=0 \text{ è pto angoloso}$$

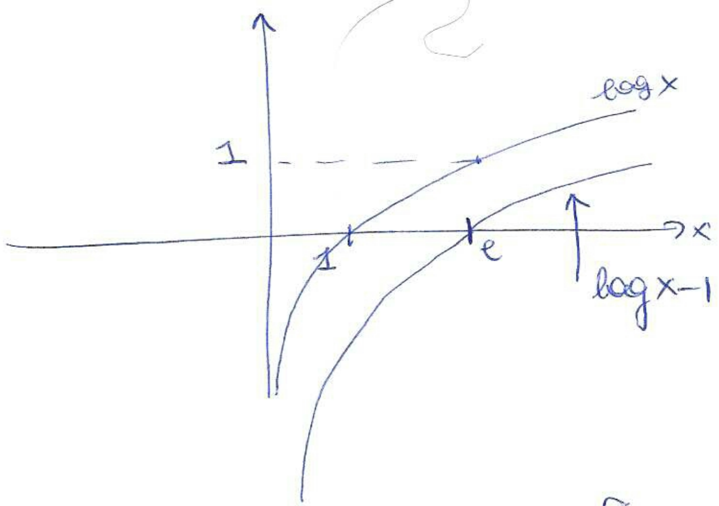


$\boxed{x=e}$ In questo caso preferisco calcolare $f'_\pm(e)$ sfruttando il limite della derivata e non il limite del rapporto incrementale, perché con il rapporto incrementale avrei $(x-e)$ al denominatore da gestire.

$$f'_+(e)=\lim_{x\to e^+}f'(x)=\lim_{x\to e^+}\left[\frac{1}{2\sqrt{\underbrace{x}_{e^+}\underbrace{|\log x-1|}_{0^+}}}\cdot\left(\underbrace{|\log x-1|}_{0^+}+\underbrace{\boxed{\frac{|\log x-1|}{\log x-1}}}_{=+1}\right)\right]$$

$$=+\infty\cdot(0+1)=+\infty$$

perché (altro foglio)



per $x \to e^+$

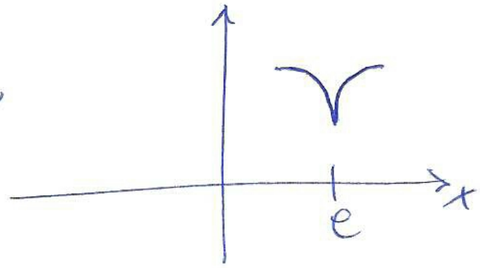
$\log x - 1 \to 0^+$

$\Rightarrow |\log x - 1| = \log x - 1$

$$f'_-(e) = \lim_{x\to e^-} f'(x) = \lim_{x\to e^-}\left[\frac{1}{2\sqrt{\underset{\downarrow\, e^-}{x}\,\underbrace{|\log x-1|}_{\downarrow\, 0^+}}}\cdot\left(\underbrace{|\log x-1|}_{\downarrow\, 0^+} + \underbrace{\boxed{\frac{|\log x-1|}{\log x-1}}}_{\| \; -1}\right)\right]$$

$$= +\infty\,(0-1) = -\infty$$

$\Rightarrow$ "e" è un pto di cuspide



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6) sia $\alpha \in \mathbb{R}$

$$\lim_{x\to 0^+} \frac{\cos x - e^x + x^{7\alpha}}{\sin x} = \lim_{x\to 0} \frac{\cancel{1} - \frac{x^2}{2} + o(x^3) - \cancel{1} - x - \frac{x^2}{2} + o(x^2) + x^{7\alpha}}{x + o(x)} =$$

$$= \lim_{x\to 0^+} \frac{-x + o(x) + x^{7\alpha}}{x} = -1 + o(1) + \lim_{x\to 0^+} x^{7\alpha - 1} =$$

$$= -1 + \begin{cases} 1 & \text{se } 7\alpha - 1 = 0 \iff \alpha = \frac{1}{7} \\ 0 & \text{se } 7\alpha - 1 > 0 \iff \alpha > \frac{1}{7} \\ +\infty & \text{se } 7\alpha - 1 < 0 \iff \alpha < \frac{1}{7} \end{cases}$$

$$= \begin{cases} 0 & \text{per } \alpha = 1/7 \\ -1 & \text{per } \alpha > 1/7 \\ +\infty & \text{per } \alpha < \frac{1}{7} \end{cases}$$

7) $$\int_0^{\pi/4} \sin^2(2x)\underbrace{\cos(2x)\, e^{\sin(2x)}\, dx}_{=\frac{dt}{2}} =$$

sostituzione: $t = \sin(2x)$ $\quad$ se $x=0 \Rightarrow t=0$

$dt = 2\cos(2x)\,dx$ $\quad$ se $x = \frac{\pi}{4} = t = \sin\left(\frac{\pi}{2}\right) = 1$

$$= \int_0^1 t^2 \cdot \frac{1}{2} e^t\, dt = \frac{1}{2}\int_0^1 \underset{f}{t^2}\, \underset{g'}{e^t}\, dt = \text{ per parti}$$

$f' = 2t$, $g = e^t$

$$= \frac{1}{2}\left[\left[t^2 e^t\right]_0^1 - \int_0^1 \underset{f}{2t}\, \underset{g'}{e^t}\, dt\right] = \frac{1}{2}\left[\left[t^2 e^t\right]_0^1 - \left(\left[2t e^t\right]_0^1 - 2\int_0^1 e^t\, dt\right)\right] =$$

$f = 2$ $\quad g = e^t$

$$= \frac{1}{2}\left[t^2 e^t - 2te^t + 2e^t\right]_0^1 = \frac{1}{2}\left(e - \cancel{2e} + \cancel{2e} - 2\right) = \frac{e-2}{2}$$

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8) ? $y: \mathbb{R}^+ \to \mathbb{R}^+$ di $\begin{cases} y' = \left(\frac{1}{x+2} + \frac{1}{(x^2+1)\arctan x}\right) y \\ y(1) = \frac{3}{4}\pi \end{cases}$

È un'eq. a variabili separabili.
Un integrale singolare è $y=0$, ma questo non soddisfa la cond. iniziale, allora la escludiamo.

$$\int \frac{dy}{y} = \int \left(\frac{1}{x+2} + \frac{1}{(1+x^2)\operatorname{arctg} x}\right) dx$$

$$\int \frac{dy}{y} = \log|y| + c = \log y + c$$: os che richiesto $y: \mathbb{R}^+ \to \mathbb{R}^+$ $\Rightarrow y > 0$.

$$\int \left(\frac{1}{x+2} + \frac{1}{(x^2+1)\operatorname{arctg} x}\right) dx = \int \frac{1}{x+2}\, dx + \boxed{\int \frac{1}{(x^2+1)\operatorname{arctg} x}\, dx} =$$

$t = \operatorname{arctg} x$

$dt = \frac{1}{x^2+1}\, dx$

$= \log|x+2| + \log|\operatorname{arctg} x| + c$

$= \log(x+2) + \log(\operatorname{arctg} x) + c$

(elimino il modulo perché si cerca $y: \underset{x}{\overset{\cup}{\mathbb{R}^+}} \to \mathbb{R}^+$)

$$\int \frac{1}{(x^2+1)\operatorname{arctg} x}\,dx =$$

$t = \operatorname{arctg} x$

$dt = \frac{1}{x^2+1}\,dx$

$$= \int \frac{dt}{t}$$

$= \log|t| = \log|\operatorname{arctg} x|$

$\Rightarrow$ ho $\log y = \log(x+2) + \log(\operatorname{arctg} x) + c$

$\log y = \log\left[(x+2)\cdot \operatorname{arctg} x\right] + c$

$e^{\log y} = e^{\log\left[(x+2)\cdot\operatorname{arctg} x\right] + c} = e^{\log[\;\;]} \cdot e^{c}$

$y(x) = (x+2)\operatorname{arctg} x \cdot e^{c}$

$\frac{3}{4}\pi = y(1) = 3\cdot \operatorname{arctg} 1 \cdot e^{c} = 3\cdot\frac{\pi}{4} e^{c} \Rightarrow e^{c} = 1$

$\Rightarrow c = 0$

$\Rightarrow y(x) = (x+2)\operatorname{arctg} x$.