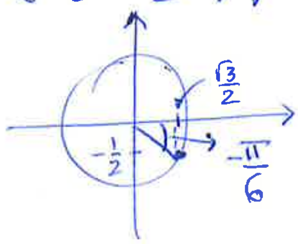
Home work (terza settimana) 5 ottobre 2018

Risultati e tracce degli esercizi

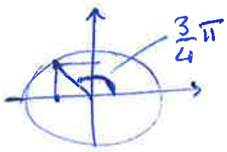
1 – $(2\sqrt{3}-2i)^5 = \left(4\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)\right)^5 = \left(4\cdot e^{-\frac{\pi}{6}i}\right)^5 = 2^{10}\cdot e^{-\frac{5}{6}\pi i} =$



$= 2^{10}\left(-\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)$

$= 2^{9}(-\sqrt{3}-i)$

– $\left(-\frac{1}{2}+\frac{1}{2}i\right)^7 = \left(\frac{1}{\sqrt{2}}\left(-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i\right)\right)^7 = 2^{-7/2}\cdot\left(e^{\frac{3}{4}\pi i}\right)^7 = 2^{-7/2}\cdot e^{\frac{21}{4}\pi i} =$



$= 2^{-7/2}\cdot e^{5/4\pi i} = 2^{-7/2}\left(-\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}i\right)$

$= -\frac{1}{16}(1+i)$

– i^i, trasformo i (alla base) in forma esponenziale $\rho e^{i\theta}$ con $-\pi<\theta\leq\pi$: $i^i = (e^{i\pi/2})^i = e^{-\pi/2}$

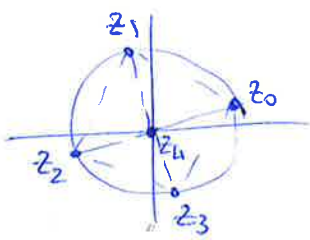
2 – $|e^{3+400i}| = e^3$ (sfrutto la proprietà $|e^z| = e^{\mathrm{Re}\,z}$)

$|e^{-1/2+\sqrt{3}/2\, i}| = e^{-1/2}$

$|e^{\pi/2+\frac{4}{3}\pi i}| = e^{\pi/2}$

3 – $(z^4-i)\cdot|z| = 0$ $\longrightarrow$ $z_4 = 0$

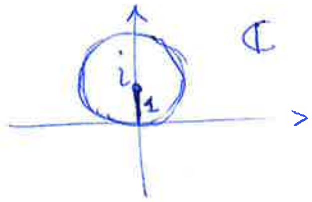
$z^4 = i$ o $|z| = 0$



calcolo radici n-sime con $\rho = 1$ $\theta = \frac{\pi}{2}$

$z_0 = e^{i\pi/8}$, $z_1 = e^{i5/8\pi}$, $z_2 = e^{i9/8\pi}$, $z_3 = e^{i\frac{13}{8}\pi}$

- $|z - e^{i\pi/2}| = 1$



trasformo $e^{i\pi/2} = i$

$\equiv |z - i| = 1 \equiv$ circonferenza di centro $(0,1)$ e raggio 1.

- $z^3 = \underbrace{\frac{\sqrt{3}}{2} + \frac{1}{2} i}_{w} \quad w = e^{i\pi/6} \quad (\rho e^{i\theta})$

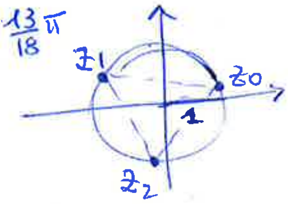
$\Rightarrow \begin{cases} n = 3 \\ \rho = 1 \quad \theta = \frac{\pi}{6} \end{cases}$

$z_k = \sqrt[n]{\rho} \cdot e^{i\varphi_k}, \quad \varphi_k = \frac{\theta + 2k\pi}{n}$

$\varphi_0 = \frac{\pi}{18}, \varphi_1 = \frac{13}{18}\pi, \varphi_2 = \frac{25}{18}\pi$

$z_0 = e^{i\pi/18}, \quad z_1 = e^{i\frac{13}{18}\pi}$

$z_2 = e^{i\frac{25}{18}\pi}$



- $z^6 = \underbrace{-i}_{w} \quad w = e^{-\frac{\pi}{2} i} \quad (\rho e^{i\theta})$

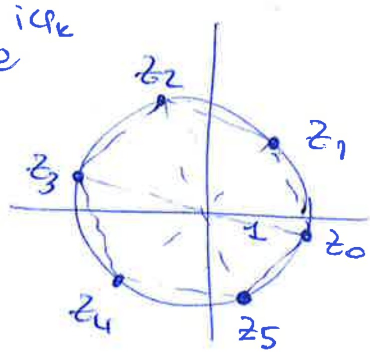
$\begin{cases} n = 6 \\ \rho = 1 \\ \theta = -\pi/2 \end{cases} \rightarrow r = \sqrt[n]{\rho}$

$\varphi_k = \frac{\theta + 2k\pi}{n}$

$\varphi_0 = -\frac{\pi}{12}, \varphi_1 = \frac{3}{12}\pi, \varphi_2 = \frac{7}{12}\pi, \varphi_3 = \frac{11}{12}\pi$

$\varphi_4 = \frac{15}{12}\pi, \varphi_5 = \frac{19}{12}\pi$

$z_0 = e^{-\pi/12 i}, \quad z_1 = e^{3/12 \pi i}, \quad z_2 = e^{\frac{7}{12}\pi i}, \ldots \quad z_k = e^{i\varphi_k}$



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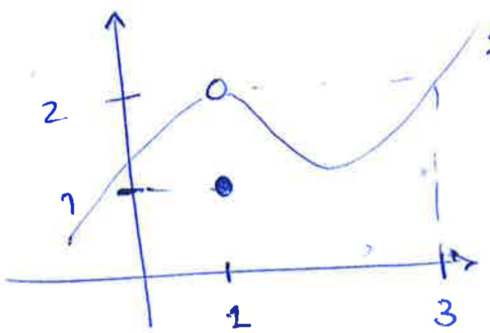
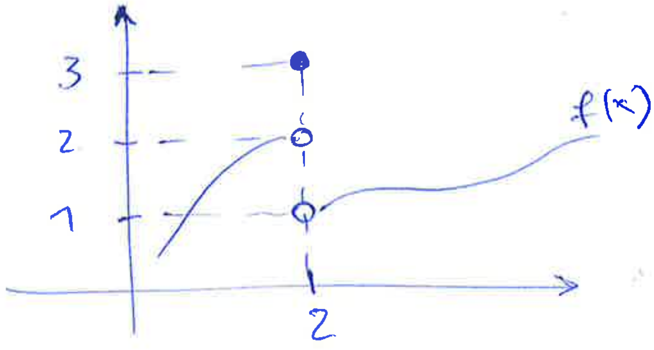
4)



$\lim_{x \to 1} f(x) = 2$

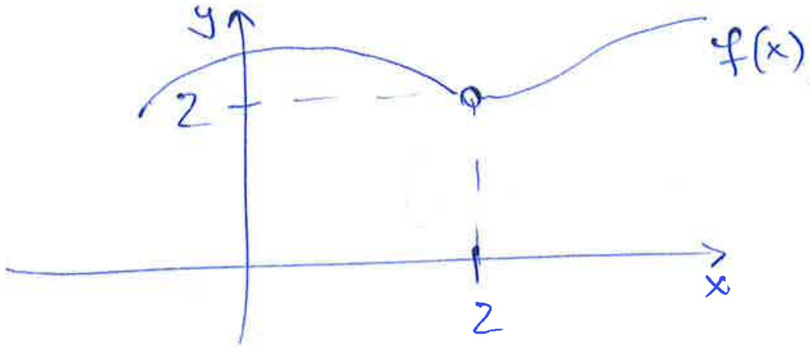
$\lim_{x \to 3} f(x) = 2$



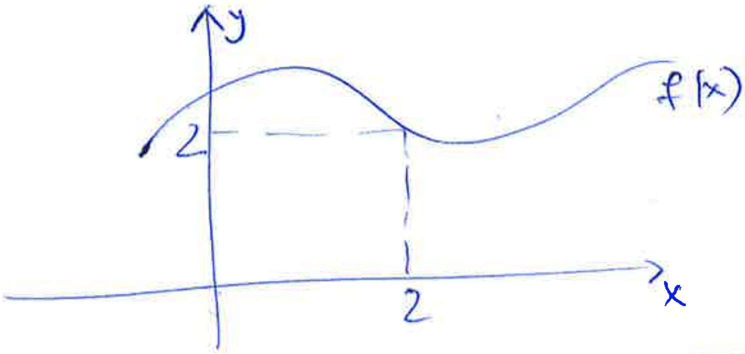
$$\lim_{x\to 2^-} f(x) = 2$$

$$\lim_{x\to 2^+} f(x) = 1$$

$$\nexists \lim_{x\to 2} f(x).$$



$$\lim_{x\to 2} f(x) = 2$$



$$\lim_{x\to 2} f(x) = 2$$

5) ? Insieme dei punti di accumulazione del dom di $f(x)$. Denoto con A tale insieme.

- $f(x) = \frac{3x}{x-1}$: $\text{dom}(f) = (-\infty, 1) \cup (1, +\infty)$, $A = \overline{\mathbb{R}}$
- $f(x) = \frac{3x}{\sqrt{x-1}}$, $\text{dom}(f) = (1, +\infty)$, $A = [1, +\infty]$
- $f(x) = \frac{\sin x}{x^2}$, $\text{dom}(f) = (-\infty, 0) \cup (0, +\infty)$, $A = \overline{\mathbb{R}}$
- $f(x) = \arcsin(x+1)$, $\text{dom}(f) = [-2, 0]$, $A = [-2, 0]$

- $f(x) = \dfrac{|x^2-3|}{\log x}$, $\text{dom}(f) = (0,1) \cup (1,+\infty)$

  $A = [0,+\infty]$

- $f(x) = \log|x|$, $\text{dom}(f) = (-\infty,0) \cup (0,+\infty)$, $A = \overline{\mathbb{R}}$

- $f(x) = \text{tg}\, x$, $\text{dom}(f) = \mathbb{R} \setminus \{x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$

  $A = \overline{\mathbb{R}}$

- $f(x) = \sin\sqrt{|x^2-1|}$, $\text{dom}(f) = \mathbb{R}$, $A = \overline{\mathbb{R}}$

- $f(x) = \sin\sqrt{x^2-1}$, $\text{dom}(f) = (-\infty,-1] \cup [1,+\infty)$

  $\overline{A} = [-\infty,-1] \cup [1,+\infty]$

- $f(x) = \sin\sqrt{1-x^2}$, $\text{dom}(f) = [-1,1]$, $A = [-1,1]$

- $f(x) = \left(\log\frac{1}{x}\right)^{-1}$, $\text{dom}(f) = (0,1) \cup (1,+\infty)$.

  $A = [0,+\infty]$

- $f(x) = \text{atan}\left(\dfrac{2x}{\cos x}\right)$, $\text{dom}(f) = \mathbb{R} \setminus \{x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$

  $A = \overline{\mathbb{R}}$

- $f(x) = e^{1/x}$, $\text{dom}(f) = (-\infty,0) \cup (0,+\infty)$, $A = \overline{\mathbb{R}}$

- $f(x) = x^x$, $\text{dom}(f) = (0,+\infty)$, $A = [0,+\infty]$

- $f(x) = \dfrac{e^x}{\log x}$, $\text{dom}(f) = (0,1) \cup (1,+\infty)$

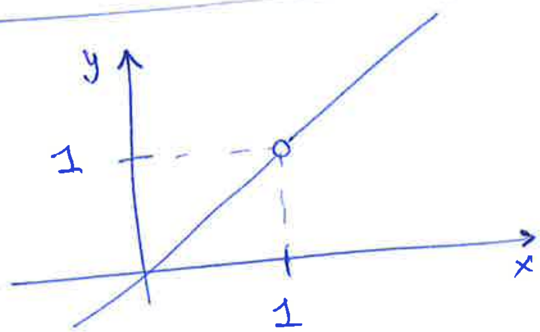
  $A = [0,+\infty]$

6 - $\lim_{x \to 0} f(x) = 0$, $\lim_{x \to +\infty} f(x) = 3$

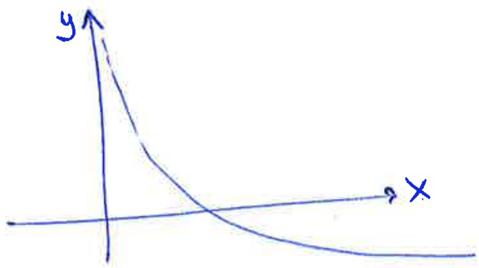
$\lim_{x \to -2} f(x)$: non ha senso calcolarlo perché -2 non è pto di acc per dom(f)

$\lim_{x \to -\infty} f(x)$: non ha senso calcolarlo perché $-\infty$ non è pto di acc per dom(f)

$\lim_{x \to 1} f(x) \sim 2.5$ (dedotto dal grafico si vede il testo)

7 - 

$f(x) = \dfrac{x^2 - x}{x - 1}$

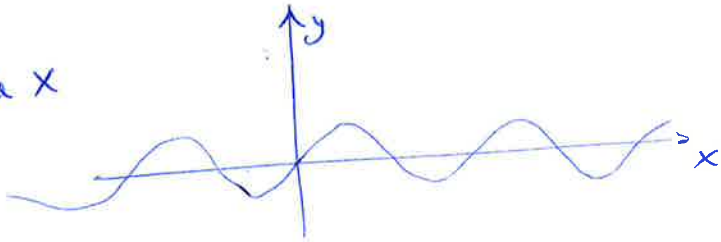
8 - 

$f(x) = -\log x$

il ~~caso~~ dom$(f) = (0, +\infty)$

e $-\infty$ non è pto di acc.

9 - $f(x) = \sin x$



10 - $\lim_{x \to 0^+} f(x)$ non esiste : $f(x) = \sin \frac{1}{x}$

o $f(x) = \cos \frac{1}{x}$ o ....

11 - $\lim_{x \to 0} f(x)$ non ha senso : $f(x) = \log(x - 1)$