Esercitazione del 28 settembre 2023

① Si consideri la funzione $f\colon \mathbb{R} \to \mathbb{R}$ definita da

$$f(x) = |x^2 - 3x + 2|.$$

Riscrivere la funzione come funzione definita a tratti e tracciarne il grafico.

② Si consideri la funzione $f\colon [0, 2\pi] \to \mathbb{R}$ definita da

$$f(x) = \left|\sin x - \frac{1}{2}\right|.$$

Riscrivere la funzione come funzione definita a tratti e tracciarne il grafico.

In ciascuno dei seguenti esercizi è data una funzione $f\colon \operatorname{dom}(f) \subseteq \mathbb{R} \to \mathbb{R}$.

- Tracciare il grafico di f tramite opportune trasformazioni applicate ai grafici delle funzioni elementari.
- Individuare il dominio e l'insieme immagine di f.

③ $f(x) = \sin^2 x$

④ $f(x) = \arcsin(2 - x)$

⑤ Verificare che

$$\log(x + e) = 1 + \log\left(1 + \frac{x}{e}\right),$$

determinando per quali $x \in \mathbb{R}$ l'uguaglianza ha significato.

⑥ Dopo aver determinato per quali $x \in \mathbb{R}$ le espressioni seguenti sono definite, scriverle sotto forma di un unico logaritmo.

(a) $\log(x^2 + 1) - 2\log x$

(b) $\log(1 + e^x) - x$

In ciascuno dei seguenti esercizi è data una funzione $f\colon \operatorname{dom}(f) \subseteq \mathbb{R} \to \mathbb{R}$. Determinare il dominio di f.

⑦ $f(x) = (\arcsin(9x^2 - 3))^{-1/3} + \sqrt[6]{\dfrac{\pi}{2} - \arccos x}$

⑧ $f(x) = \sqrt{\log_{\frac{1}{2}}(2\sin x) + \log_{\frac{1}{2}}(\sqrt{2}\cos x)}$

⑨ $f(x) = (x - 1 - \sqrt{x^2 - 2x})^{\log(e-x)}$

Conclusione dell'ultimo esercizio dell'esercitazione del 25 settembre 2023

$$f(x) = \sqrt{e^{2x} - 4e^x + 3} \cdot (\log^2 x - 1)^{-\frac{1}{2}}$$

$$\begin{cases} e^{2x} - 4e^x + 3 \geq 0 \\ \log^2 x - 1 > 0 \\ x > 0 \end{cases}$$

$$\log^2 x = (\log x)^2$$

$$(\log^2 x - 1)^{-\frac{1}{2}} = \frac{1}{(\log^2 x - 1)^{1/2}} = \frac{1}{\sqrt{\log^2 x - 1}}$$

$$e^{2x} - 4e^x + 3 \geq 0$$

$$(e^x)^2 - 4e^x + 3 \geq 0 \qquad e^x = t$$

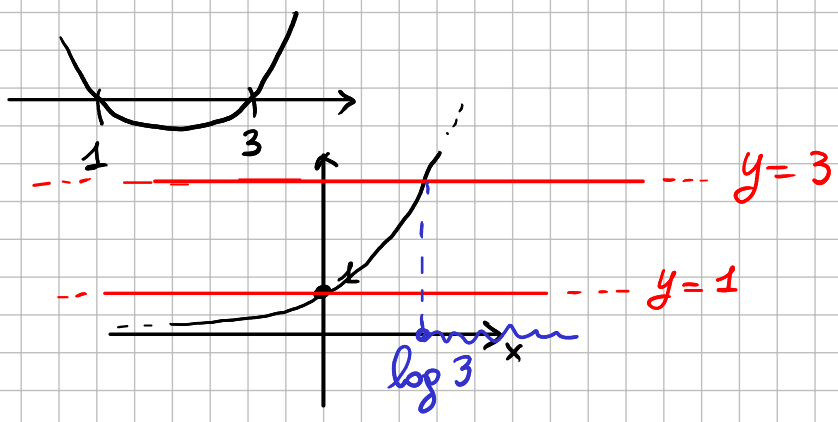
$$t^2 - 4t + 3 \geq 0$$

$$(t-1)(t-3) \geq 0$$

$$t \leq 1 \vee t \geq 3$$

$$e^x \leq 1 \vee e^x \geq 3$$

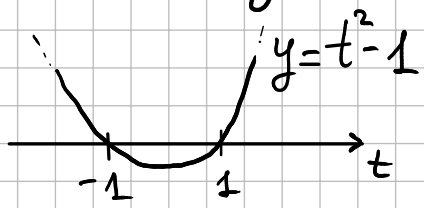
$$x \leq 0 \vee x \geq \log 3$$



$$\log^2 x - 1 > 0 \qquad t = \log x$$

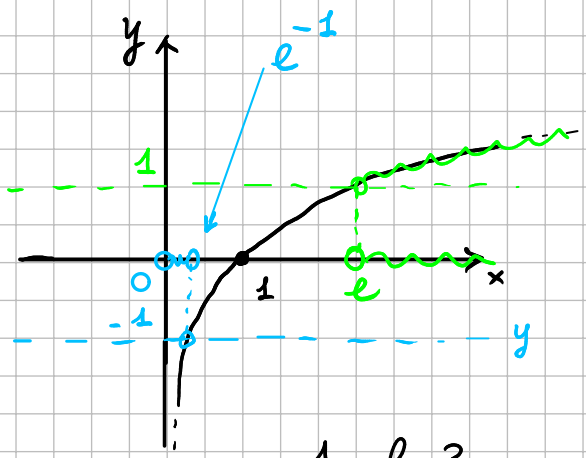
$$t^2 - 1 > 0$$

$$t < -1 \vee t > 1$$



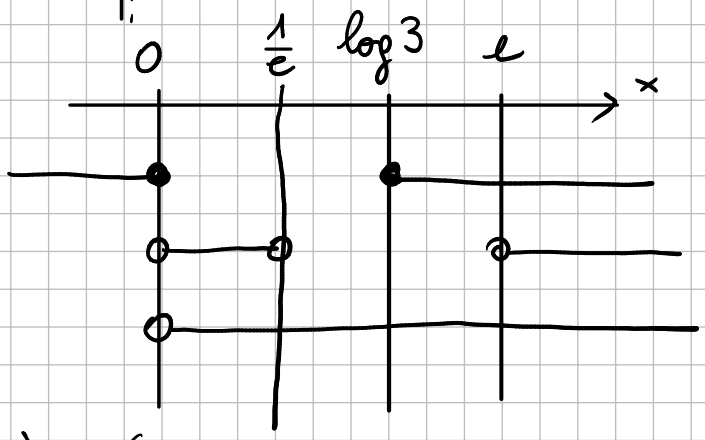
$$\log x < -1 \vee \log x > 1$$

$0 < x < \frac{1}{e} \vee \quad x > e$



$2 < e < 3$

$$\begin{cases} x \leq 0 \vee x \geq \log 3 \\ 0 < x < \frac{1}{e} \vee x > e \\ x > 0 \end{cases}$$



$$\mathrm{dom}(f) = (e, +\infty)$$

Si consideri la funzione $f\colon \mathbb{R} \to \mathbb{R}$ definita da

$$f(x) = |x^2 - 3x + 2|.$$

Riscrivere la funzione come funzione definita a tratti e tracciarne il grafico.

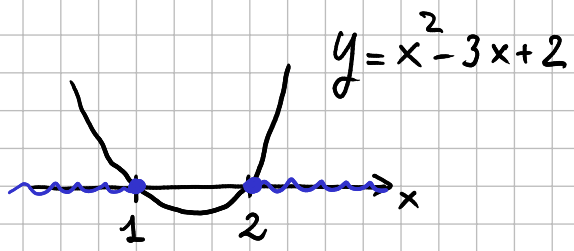
$$|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$

$\text{dom}(f) = \mathbb{R}$

$x^2 - 3x + 2 \geq 0$

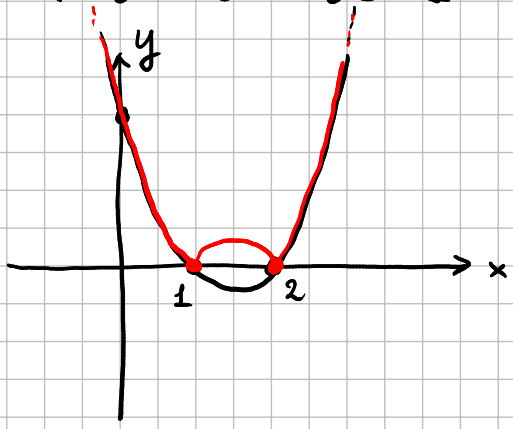
$(x-1)(x-2) \geq 0$

$x \leq 1 \vee x \geq 2$



$$f(x) = \begin{cases} x^2 - 3x + 2 & \text{se } x^2 - 3x + 2 \geq 0 \\ -(x^2 - 3x + 2) & \text{se } x^2 - 3x + 2 < 0 \end{cases}$$

$$f(x) = \begin{cases} x^2 - 3x + 2 & \text{se } x \leq 1 \vee x \geq 2 \\ -x^2 + 3x - 2 & \text{se } 1 < x < 2 \end{cases}$$



$y = x^2 - 3x + 2$

$f(x) = |x^2 - 3x + 2|$

Si consideri la funzione $f : [0, 2\pi] \to \mathbb{R}$ definita da

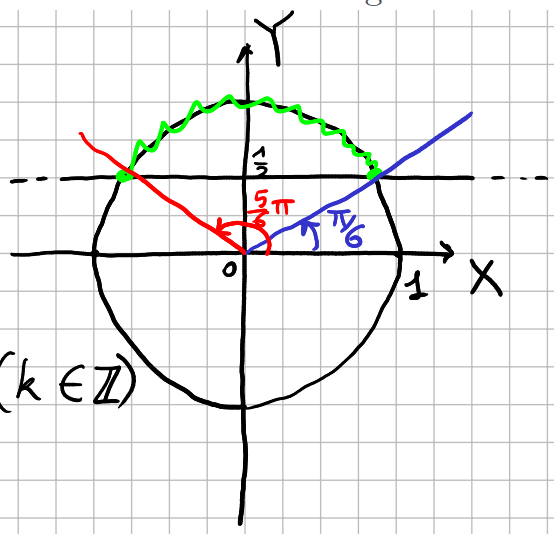
$$f(x) = \left| \sin x - \frac{1}{2} \right| .$$

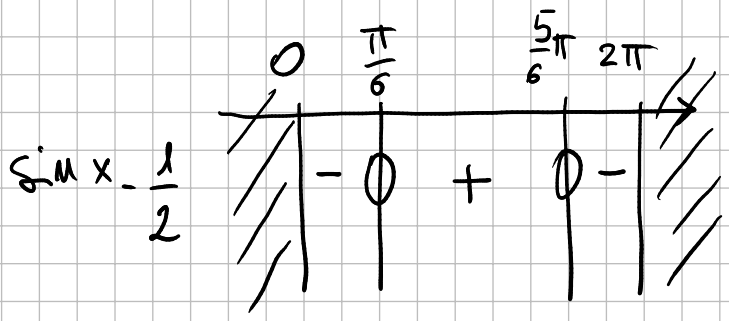
Riscrivere la funzione come funzione definita a tratti e tracciarne il grafico.

$$\sin x - \frac{1}{2} \geq 0$$

$$\sin x \geq \frac{1}{2}$$

$$\frac{\pi}{6} + 2k\pi \leq x \leq \frac{5}{6} + 2k\pi \quad (k \in \mathbb{Z})$$

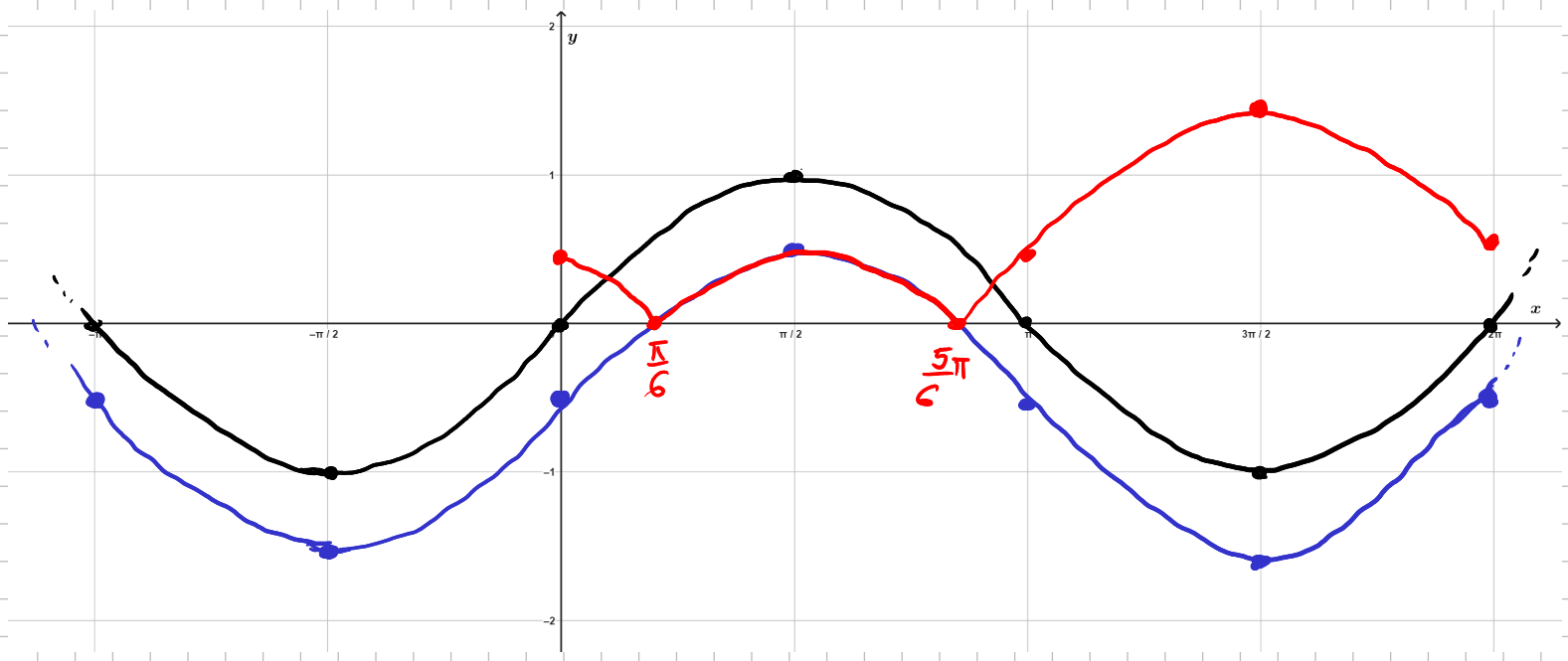




$$f : [0, 2\pi] \longrightarrow \mathbb{R}$$

$$f(x) = \left| \sin x - \frac{1}{2} \right|$$

$$f(x) = \begin{cases} \sin x - \frac{1}{2} & \text{se } \frac{\pi}{6} \leq x \leq \frac{5}{6}\pi \\ \frac{1}{2} - \sin x & \text{se } 0 \leq x < \frac{\pi}{6} \vee \frac{5}{6}\pi < x \leq 2\pi \end{cases}$$



$y = \sin x \quad (x \in \mathbb{R})$

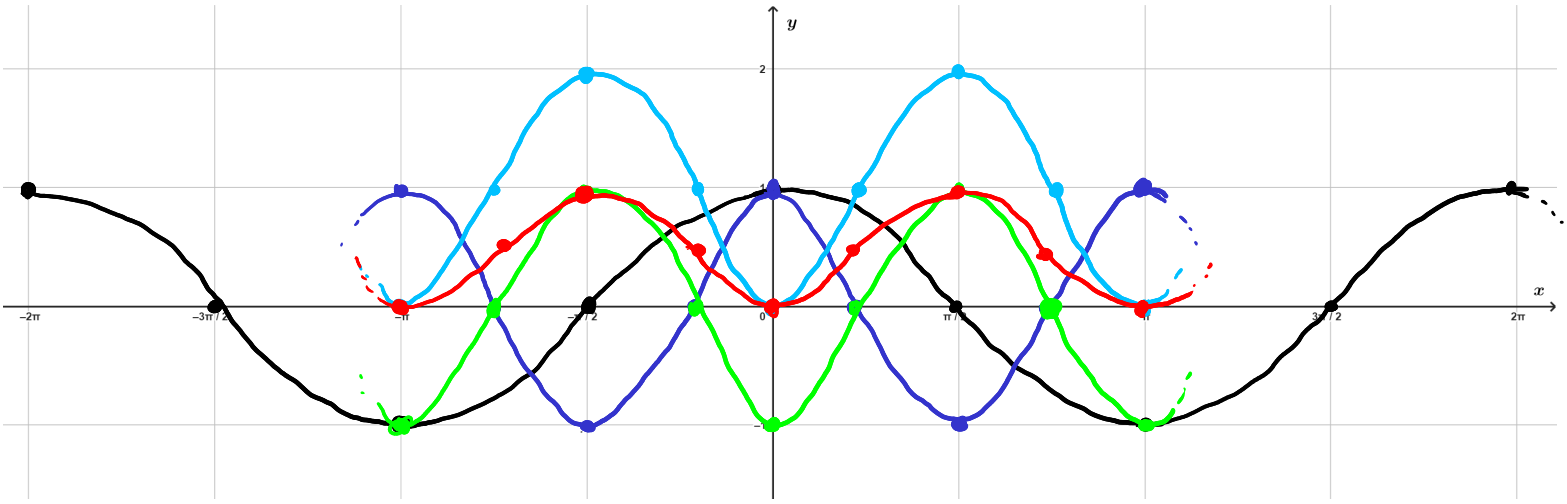
$y = \sin x - \frac{1}{2} \quad (x \in \mathbb{R})$

$y = \left|\sin x - \frac{1}{2}\right|, \quad x \in [0, 2\pi]$

$f : [0, 2\pi] \longrightarrow \mathbb{R}$

$f(x) = \left|\sin x - \frac{1}{2}\right|$

$$f(x) = \sin^2 x$$



$$\cos 2x = \cos^2 x - \sin^2 x \quad \forall x \in \mathbb{R}$$

Si può riscrivere come

$$\cos 2x = (1-\sin^2 x) - \sin^2 x$$

$$\cos 2x = 1 - 2\sin^2 x \quad \forall x \in \mathbb{R}$$

$$\sin^2 x = \frac{1}{2}(1-\cos 2x) \quad \forall x \in \mathbb{R}$$

$y = \cos x$

$y = \cos 2x$

$y = -\cos 2x$

$y = 1 - \cos 2x$

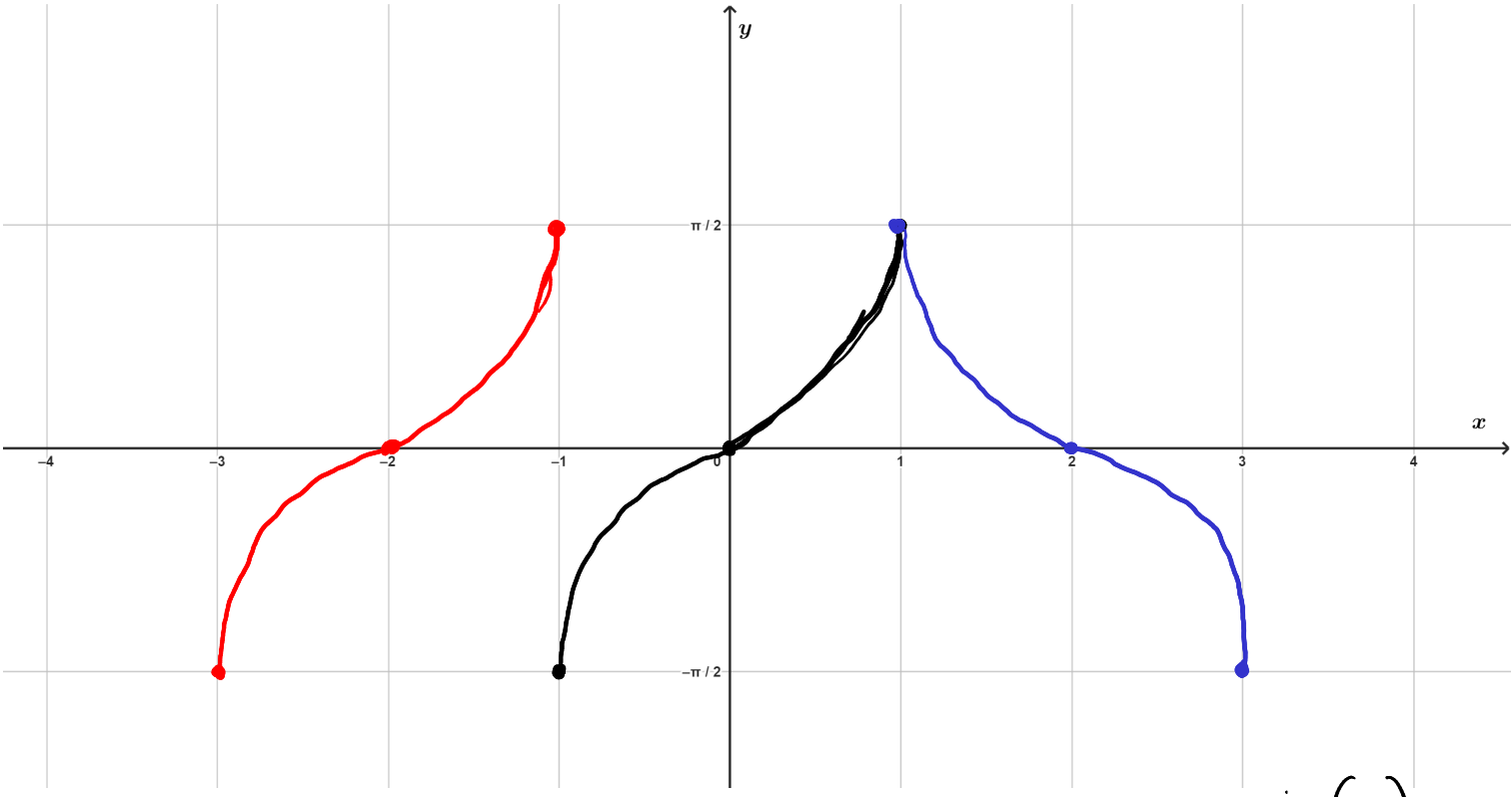
$y = \frac{1}{2}(1-\cos 2x)$

$f(x) = \sin^2 x$ ha dominio $\mathbb{R}$

$\mathrm{im}(f) = [0,1]$

La funzione ha periodo π

$$f(x) = \arcsin(2-x)$$



$$\begin{cases} x' = -x \\ y' = y \end{cases}$$

$y = \arcsin(x)$

$y = \arcsin(x+2)$

$y = \arcsin(-x+2)$

Verificare l'uguaglianza

$$\log(x+e) = 1 + \log\left(1+\frac{x}{e}\right)$$

L'uguaglianza ha significato se e solo se

$$\begin{cases} x+e>0 \\ 1+\frac{x}{e}>0 \end{cases} \iff \begin{cases} x>-e \\ \frac{x}{e}>-1 \end{cases} \iff x>-e$$

$\log(A\cdot B) = \log A + \log B$
$\forall A, B > 0$

$$\log(x+e) = \log\left(e\left(1+\frac{x}{e}\right)\right) =$$

$$= \log e + \log\left(1+\frac{x}{e}\right) =$$

$$= 1 + \log\left(1+\frac{x}{e}\right) \quad \forall x \in (-e, +\infty)$$

Scrivere sotto forma di un unico logaritmo

$$\log(x^2+1) - 2\log x$$

L'espressione ha significato se e solo se

$$\begin{cases} x^2+1>0 \ \forall x \in \mathbb{R} \\ x>0 \end{cases} \iff x>0$$

$$\log\frac{A}{B} = \log A - \log B \quad \forall A, B > 0$$

$$\log A^x = x \cdot \log A \quad \forall A>0, \forall x \in \mathbb{R}$$

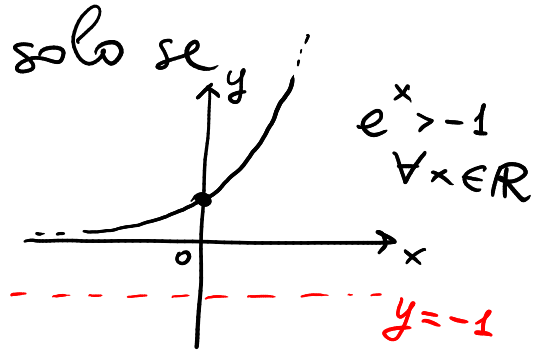
$$\log(x^2+1) - 2\log x = \log(x^2+1) - \log x^2 =$$
$$= \log\left(\frac{x^2+1}{x^2}\right) =$$
$$= \log\left(1+\frac{1}{x^2}\right) \quad \forall x \in (0,+\infty)$$

Scrivere sotto forma di un unico logaritmo

$$\log(1+e^x) - x$$

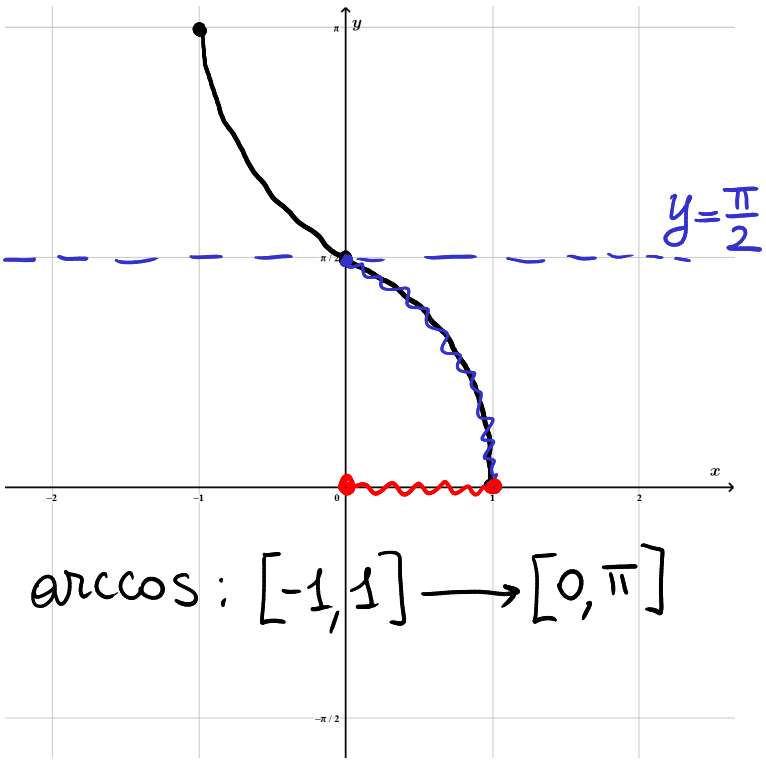
L'espressione ha significato se e solo se

$$\underbrace{1}_{>0} + \underbrace{e^x}_{>0 \ \forall x \in \mathbb{R}} > 0 \quad \forall x \in \mathbb{R}$$

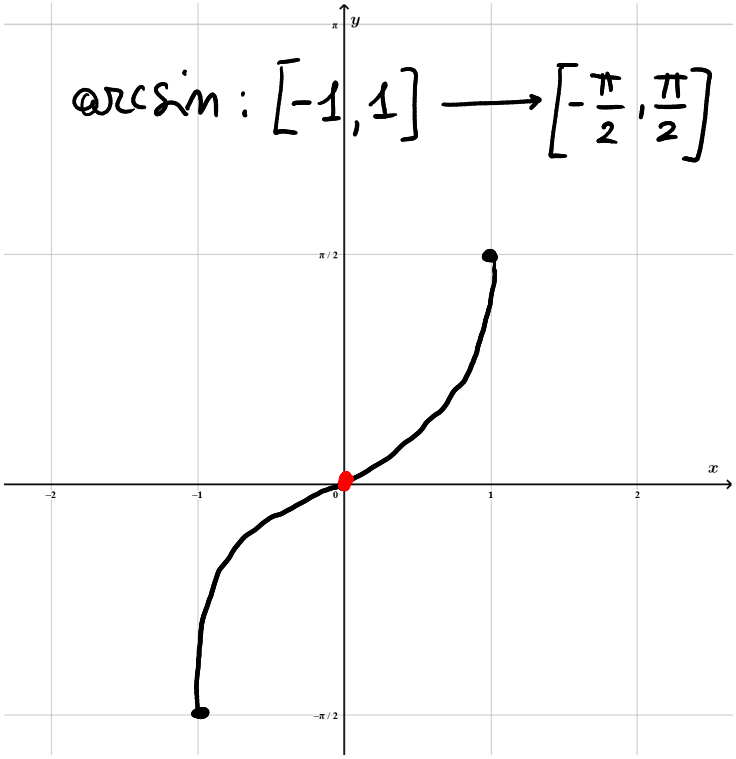


$$\log(1+e^x) - x = \log(1+e^x) - \log e^x =$$
$$= \log\left(\frac{1+e^x}{e^x}\right) =$$
$$= \log\left(1+\frac{1}{e^x}\right) =$$
$$= \log(1+e^{-x}) \quad \forall x \in \mathbb{R}$$

$$f(x) = \left(\arcsin\left(9x^2-3\right)\right)^{-1/3} + \sqrt[6]{\frac{\pi}{2} - \arccos x}$$



$\arccos : [-1,1] \longrightarrow [0,\pi]$



$\arcsin : [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$$f(x) = \frac{1}{\sqrt[3]{\arcsin\left(9x^2-3\right)}} + \sqrt[6]{\frac{\pi}{2} - \arccos x}$$

$$\begin{cases} -1 \leq 9x^2-3 \leq 1 \\ \arcsin\left(9x^2-3\right) \neq 0 \\ -1 \leq x \leq 1 \\ \frac{\pi}{2} - \arccos x \geq 0 \end{cases}$$

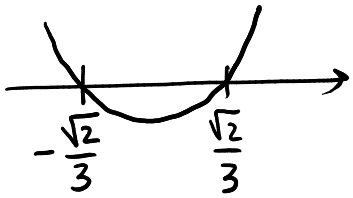
$$\frac{\pi}{2} - \arccos x \geq 0 \iff \arccos x \leq \frac{\pi}{2} \iff 0 \leq x \leq 1$$

$$\arcsin\left(9x^2-3\right) \neq 0 \iff 9x^2-3 \neq 0 \iff 9x^2 \neq 3 \iff x \neq \pm\frac{\sqrt{3}}{3}$$

$$-1 \leq 9x^2-3 \leq 1 \iff \begin{cases} 9x^2-3 \geq -1 \\ 9x^2-3 \leq 1 \end{cases} \iff \begin{cases} 9x^2-2 \geq 0 \\ 9x^2-4 \leq 0 \end{cases}$$

$$9x^2-2\geq 0$$

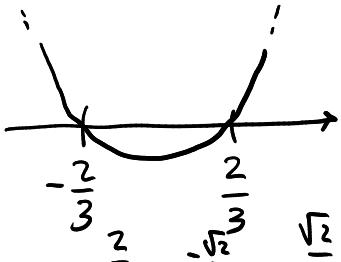
$$x\leq -\frac{\sqrt{2}}{3} \vee x\geq \frac{\sqrt{2}}{3}$$



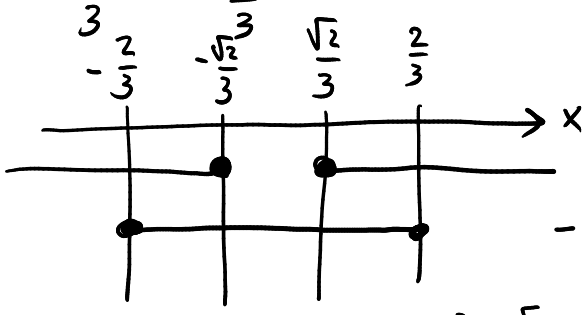
$$9x^2-2=0$$
$$9x^2=2$$
$$x^2=\frac{2}{9}$$
$$x=\pm\frac{\sqrt{2}}{3}$$

$$9x^2-4\leq 0$$

$$-\frac{2}{3}\leq x\leq \frac{2}{3}$$

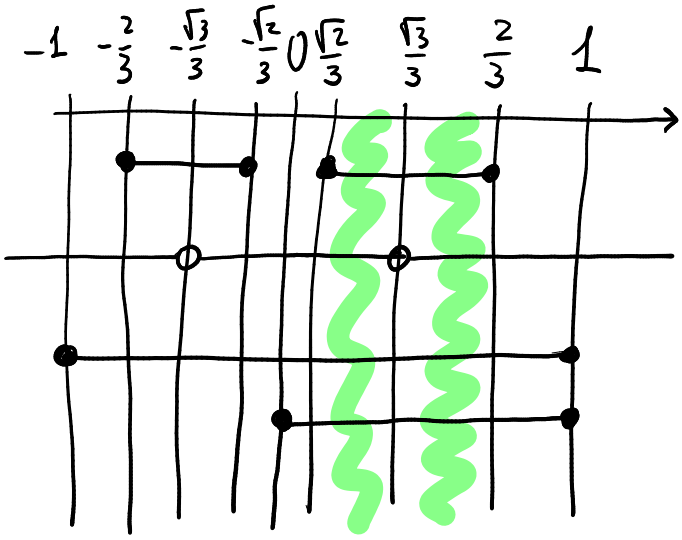


$$\begin{cases} 9x^2-2\geq 0 \\ 9x^2-4\leq 0 \end{cases}$$



$$-\frac{2}{3}\leq x\leq -\frac{\sqrt{2}}{3} \vee \frac{\sqrt{2}}{3}\leq x\leq \frac{2}{3}$$

$$\begin{cases} -\frac{2}{3}\leq x\leq -\frac{\sqrt{2}}{3} \vee \frac{\sqrt{2}}{3}\leq x\leq \frac{2}{3} \\ x\neq \pm\frac{\sqrt{3}}{3} \\ -1\leq x\leq 1 \\ 0\leq x\leq 1 \end{cases}$$

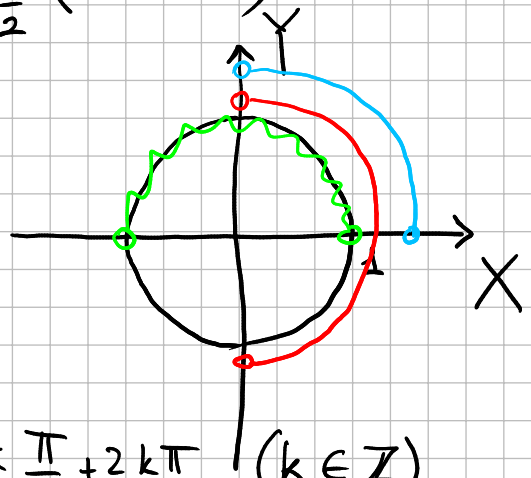


$$\mathrm{dom}(f)=\left[\frac{\sqrt{2}}{3},\frac{\sqrt{3}}{3}\right)\cup\left(\frac{\sqrt{3}}{3},\frac{\sqrt{2}}{3}\right]$$

$$f(x) = \sqrt{\log_{\frac{1}{2}}(2\sin x) + \log_{\frac{1}{2}}(\sqrt{2}\cos x)}$$

$$\begin{cases} 2\sin x > 0 \\ \sqrt{2}\cos x > 0 \\ \log_{\frac{1}{2}}(2\sin x) + \log_{\frac{1}{2}}(\sqrt{2}\cos x) \geq 0 \end{cases}$$

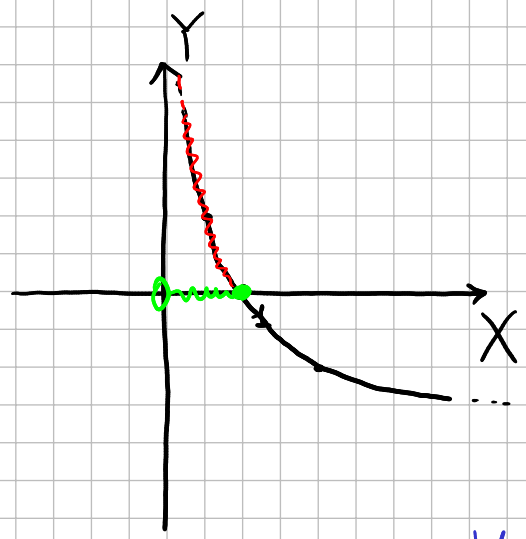
$$\begin{cases} \sin x > 0 \\ \cos x > 0 \end{cases}$$



$$\begin{cases} \sin x > 0 \\ \cos x > 0 \end{cases} \iff 2k\pi < x < \frac{\pi}{2} + 2k\pi \quad (k \in \mathbb{Z})$$

$$\log_{\frac{1}{2}}(2\sin x) + \log_{\frac{1}{2}}(\sqrt{2}\cos x) \geq 0$$

$$\log_{\frac{1}{2}}(2\sin x \cdot \sqrt{2}\cos x) \geq 0$$



$$0 < 2\sqrt{2}\sin x \cos x \leq 1$$

$$0 < \sqrt{2}\sin 2x \leq 1$$

$\sin 2x = 2\sin x \cos x \quad \forall x \in \mathbb{R}$

$$\begin{cases} \sqrt{2}\sin 2x > 0 \\ \sqrt{2}\sin 2x \leq 1 \end{cases}$$

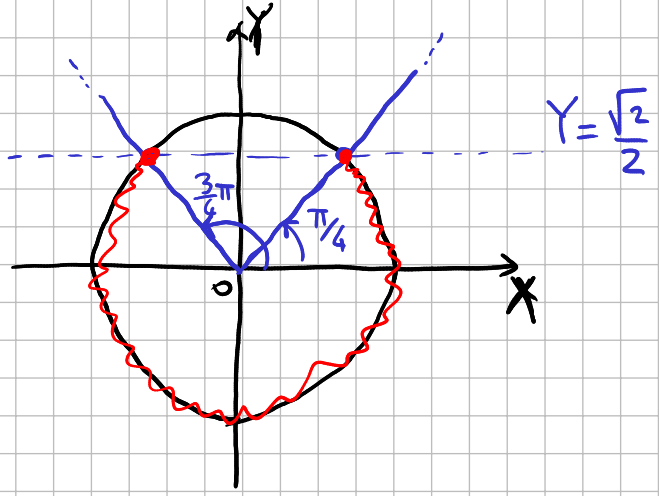
$$\sqrt{2}\sin 2x > 0 \iff \sin 2x > 0 \iff 2k\pi < 2x < \pi + 2k\pi$$

$$\iff k\pi < x < \frac{\pi}{2} + k\pi \quad (k \in \mathbb{Z})$$

$$\sqrt{2}\sin 2x \le 1$$

$$\sin 2x \le \frac{1}{\sqrt{2}}$$

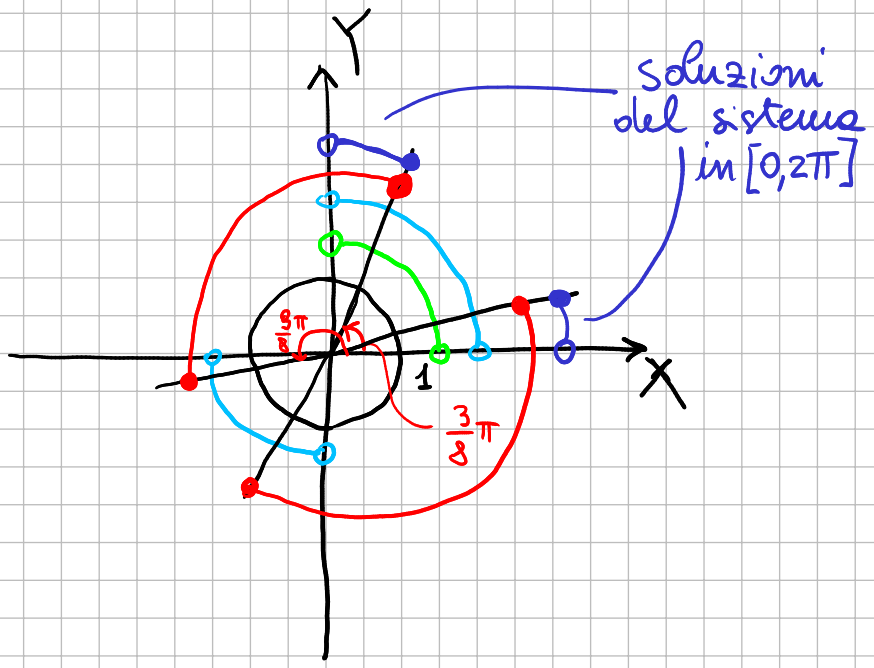
$$\sin 2x \le \frac{\sqrt{2}}{2}$$



$$\frac{3}{4}\pi + 2k\pi \le 2x \le \frac{9}{4}\pi + 2k\pi$$

$$\frac{3}{8}\pi + k\pi \le x \le \frac{9}{8}\pi + k\pi$$

$$\begin{cases} \sin x > 0 \\ \cos x > 0 \\ \sin 2x > 0 \\ \sin 2x \le \frac{\sqrt{2}}{2} \end{cases} \iff \begin{cases} 2k\pi < x < \frac{\pi}{2} + 2k\pi \\ k\pi < x < \frac{\pi}{2} + k\pi \\ \frac{3}{8}\pi + k\pi \le x \le \frac{9}{8}\pi + k\pi \end{cases}$$



$\frac{\pi}{8}$ radianti

$180° : 8 = 22{,}5°$

$$\mathrm{dom}(f) = \left\{ x \in \mathbb{R} \;\middle|\; 2k\pi < x \le \frac{\pi}{8} + 2k\pi \ \vee\ \frac{3}{8}\pi + 2k\pi \le x < \frac{\pi}{2} + 2k\pi,\ k \in \mathbb{Z} \right\}$$

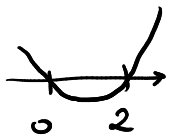
$$f(x) = \left(x - 1 - \sqrt{x^2 - 2x}\right)^{\log(e-x)}$$

$$\begin{cases} x - 1 - \sqrt{x^2-2x} > 0 \\ x^2 - 2x \geq 0 \\ e - x > 0 \end{cases}$$ perchè il logaritmo sia definito

$$e - x > 0 \iff x < e$$

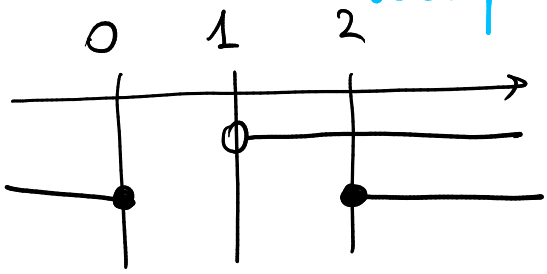
$$\begin{cases} x - 1 - \sqrt{x^2-2x} > 0 \\ x^2 - 2x \geq 0 \end{cases} \iff \begin{cases} \sqrt{x^2-2x} < x - 1 \\ x^2 - 2x \geq 0 \end{cases}$$

$$\iff \begin{cases} \sqrt{x^2-2x} < x - 1 \\ x - 1 > 0 \\ x^2 - 2x \geq 0 \end{cases} \iff \begin{cases} x^2 - 2x < (x-1)^2 \\ x > 1 \\ x(x-2) \geq 0 \end{cases}$$



$$\iff \begin{cases} \cancel{x^2} - \cancel{2x} < \cancel{x^2} - \cancel{2x} + 1 \\ x > 1 \\ x \leq 0 \vee x \geq 2 \end{cases} \iff \begin{cases} x > 1 \\ x \leq 0 \vee x \geq 2 \end{cases}$$

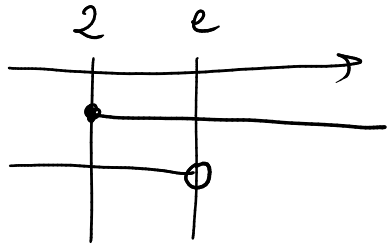
La prima disequazione è sempre verificata



$x \geq 2$ ~ soluzioni della disequazione irrazionale

$$\begin{cases} x - 1 - \sqrt{x^2-2x} > 0 \\ x^2 - 2x \geq 0 \end{cases}$$

$$\begin{cases} x-1-\sqrt{x^2-2x} \geq 0 \\ x^2-2x \geq 0 \\ e-x>0 \end{cases} \iff \begin{cases} x \geq 2 \\ x<e \end{cases}$$



$2 \leq x < e$

$$\mathrm{dom}(f) = [2, e)$$