

$$\int_0^1 \sqrt{1-y^2} dy =$$

12/12/2023

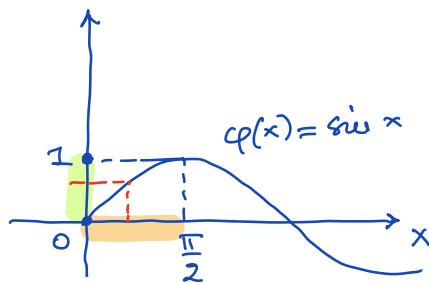
$$\int_a^b f(y) dy = \int_{\varphi'(a)}^{\varphi'(b)} f(\varphi(x)) \varphi'(x) dx$$

$$y = \varphi(x)$$

$$f(y) = \sqrt{1-y^2}$$

$$y = \sin x = \varphi(x)$$

$$dy = \varphi'(x) dx = \cos x \cdot dx$$



$\sin x$ non è biiettiva (o bivoca)
ma tutto \mathbb{R} , ma lo è su $[0, \frac{\pi}{2}]$
 $\Rightarrow \exists \varphi'(x)$

$$\text{se } y = 0 \Rightarrow \varphi^{-1}(0) = 0 = x$$

$$\text{se } y = 1 \Rightarrow \varphi^{-1}(1) = \frac{\pi}{2} = x$$

$$= \int_0^{\frac{\pi}{2}} \sqrt{1-\sin^2 x} \cdot \cos x dx = \int_0^{\frac{\pi}{2}} \cos^2 x dx =$$

$$1 - \sin^2 x = \cos^2 x \Rightarrow \sqrt{1 - \sin^2 x} = \sqrt{\cos^2 x} = |\cos x| =$$

$\cos x$

perché $0 \leq x \leq \frac{\pi}{2} \Rightarrow \cos x \geq 0$ e $|\cos x| = \cos x$

$$= \int_0^{\frac{\pi}{2}} 1 - \sin^2 x dx \stackrel{\text{line}}{=} \int_0^{\frac{\pi}{2}} 1 dx - \int_0^{\frac{\pi}{2}} \sin^2 x dx =$$

ricordo che una primitiva di $\sin^2 x$ è

$$\frac{-\sin x \cdot \cos x + x}{2}$$

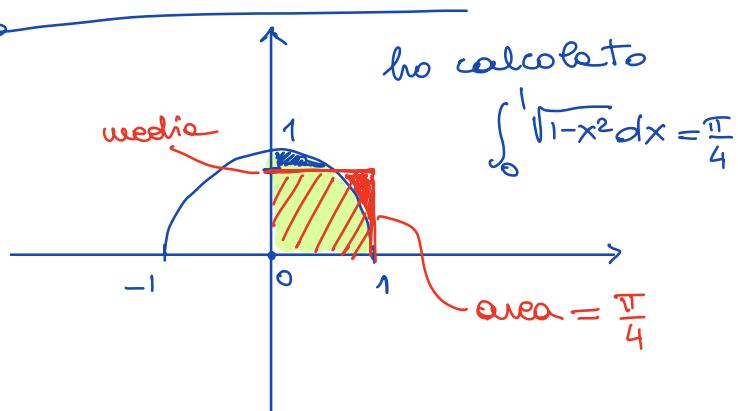
$$= \left[x \right]_0^{\pi/2} - \left[\frac{-\sin x \cdot \cos x + x}{2} \right]_0^{\pi/2} =$$

$$= \frac{\pi}{2} - \left[\frac{\pi}{4} - 0 \right] = \frac{\pi}{4}$$

$$y = f(x) = \sqrt{1-x^2}$$

$$x^2 + y^2 = 1$$

$$y^2 = 1 - x^2$$



$y = \pm \sqrt{1-x^2}$ non è una funzione

$y = +\sqrt{1-x^2}$ semicrf sup

$y = -\sqrt{1-x^2}$ semicrf inf

? $\int_0^1 \sqrt{1-x^2} dx =$

$$\int_a^b f(x) dx = \frac{1}{b-a} \int_a^b f(x) dx$$

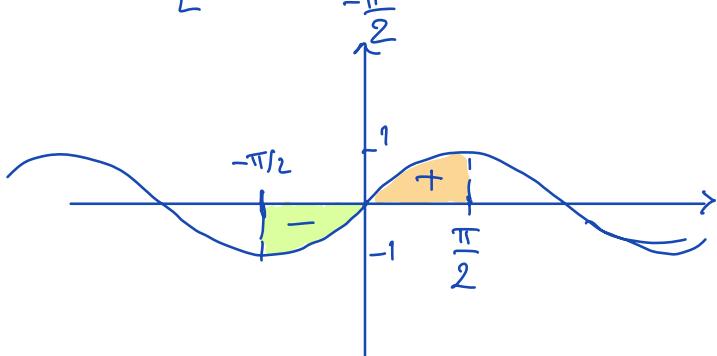
$$= \frac{1}{1} \int_0^1 \sqrt{1-x^2} dx = \frac{\pi}{4} = \text{media integrale}$$

~ 0.78

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (\sin x + x^3) dx$$

$$f(x) = \sin x + x^3 \text{ è dispari} \Rightarrow \int = 0$$

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x dx = \left[-\cos x \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = -\cos\left(\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) = 0$$



? Área del trapezio inverso a $f(x) = x \sin x$
 in $[\frac{\pi}{2}, 2\pi]$

$$A(\mathcal{T}(f; a, b)) = \int_a^b |f(x)| dx$$

$$\text{dove } f(x) \geq 0 \Rightarrow |f(x)| = f(x)$$

$$\text{dove } f(x) < 0 \Rightarrow |f(x)| = -f(x)$$

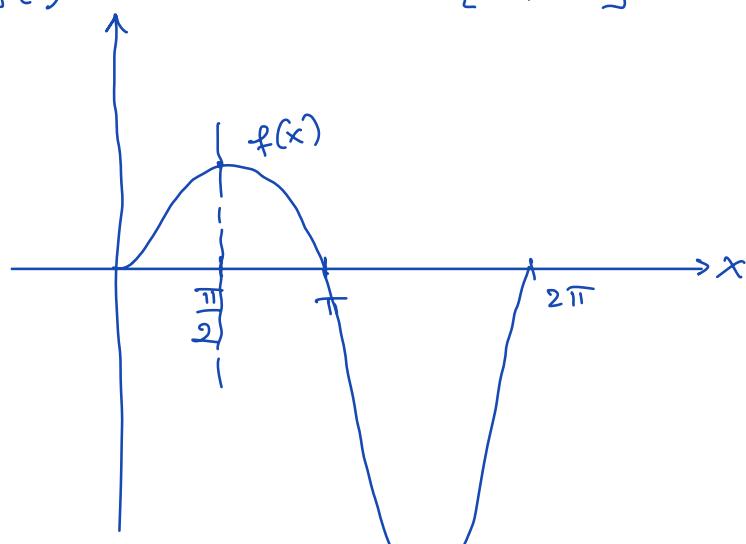
$$? x \in [\frac{\pi}{2}, 2\pi] : f(x) \geq 0, x \sin x \geq 0$$

$$F_1: x > 0 \quad \text{und} \quad x \in [\frac{\pi}{2}, 2\pi]$$

F_2 sin $x > 0$ $\frac{\pi}{2} \leq x \leq \pi$ null in $\text{fertig } [\frac{\pi}{2}, 2\pi]$

	$\frac{\pi}{2}$	π	2π
F_1	+		+
F_2	+	-	
f	+	-	

$$|f(x)| = \begin{cases} f(x) = x \cdot \sin x & \text{in } [\frac{\pi}{2}, \pi] \\ -f(x) = -x \cdot \sin x & \text{in } [\pi, 2\pi] \end{cases}$$



$$\begin{aligned} A &= \int_{\frac{\pi}{2}}^{2\pi} |f(x)| dx = \int_{\frac{\pi}{2}}^{\pi} f(x) dx + \int_{\pi}^{2\pi} -f(x) dx = \\ &= \int_{\frac{\pi}{2}}^{\pi} x \sin x dx - \int_{\pi}^{2\pi} x \sin x dx = \textcircled{*} \end{aligned}$$

cerco una primitiva $G(x)$ di $x \cdot \sin x$
e poi applico il 2°thm fondam. del calcolo

$$\int x \cdot \sin x dx = \text{per parti} \quad f'g = fg - \int fg'$$

$f'(x) = \sin x \quad g(x) = x$

$f(x) = -\cos x \quad g'(x) = 1$

$$= -x \cdot \cos x - \int -\cos x dx = -x \cos x + \sin x = G(x)$$

$$\textcircled{*} = \left[-x \cos x + \sin x \right]_{\frac{\pi}{2}}^{\pi} - \left[-x \cos x + \sin x \right]_{\frac{\pi}{2}}^{2\pi}$$

$$= -\pi \cdot \underbrace{\cos \pi}_{(-1)} + \underbrace{\sin \pi}_0 - \left(-\frac{\pi}{2} \cos \frac{\pi}{2} + \underbrace{\sin \frac{\pi}{2}}_1 \right) - \left[-2\pi \cdot \underbrace{\cos 2\pi}_1 + \underbrace{\sin 2\pi}_0 - \left(-\pi \cos \pi \underbrace{\cos \pi}_{(-1)} + \underbrace{\sin \pi}_0 \right) \right]$$

$$= \pi - 1 - [-2\pi - \pi] = 4\pi - 1$$

$$\bullet \int x \cdot \log x dx = \text{P.P.} \quad f'g = fg - \int fg'$$

e cerco primitiva $G(x)$ di $x \cdot \log x$

$$\int x \cdot \log x dx = \begin{aligned} f'(x) &= x & g(x) &= \log x \\ f(x) &= \frac{1}{2}x^2 & g'(x) &= \frac{1}{x} \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2}x^2 \log x - \int \frac{1}{2}x^2 \cdot \frac{1}{x} dx = \frac{1}{2}x^2 \log x - \frac{1}{2} \cdot \frac{1}{2}x^2 \\
 G(x) &= \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 \\
 \Rightarrow &= \left[\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 \right] e^2 = \log e = 1 \\
 &= \frac{1}{2}e^4 \log e^2 - \frac{1}{4}e^4 - \left(\frac{1}{2}e^2 \log e - \frac{1}{4}e^2 \right) \\
 &= \frac{1}{2}e^4 \cdot 2 - \frac{1}{4}e^4 - \left(\frac{1}{2}e^2 - \frac{1}{4}e^2 \right) = \\
 &= \frac{3}{4}e^4 - \frac{1}{4}e^2
 \end{aligned}$$

$$\begin{aligned}
 \int_0^{\pi/2} \frac{\sin(2x)}{1+4\cos^2 x} dx &= y = \varphi(x) = 1+4\cos^2 x \\
 dy &= \varphi'(x)dx = 4 \cdot 2\cos x \cdot (-\sin x) dx
 \end{aligned}$$

$$\begin{aligned}
 &= -\frac{1}{4} \int_0^{\pi/2} \frac{-4\sin(2x) dx}{1+4\cos^2 x} = -4 \sin(2x) dx \\
 &= -\frac{1}{4} \int_5^1 \frac{1}{y} dy = -\frac{1}{4} \left[\log |y| \right]_5^1 = \\
 &= -\frac{1}{4} \left[\underbrace{\log 1}_{0} - \log 5 \right] = \frac{1}{4} \log 5
 \end{aligned}$$

$$\int_1^2 \sqrt{x} \cdot \arctan(\sqrt{x^3}) dx = \sqrt{x^3} = x^{3/2}$$

$$y = \varphi(x) = \sqrt{x^3}$$

$$dy = \varphi'(x)dx = \frac{3}{2} \sqrt{x} dx$$

$$\text{se } x = 1 \Rightarrow y = \sqrt{1^3} = 1$$

$$\text{se } x = 2 \Rightarrow y = \sqrt{2^3} = 2\sqrt{2}$$

$$= \frac{2}{3} \int_1^{2\sqrt{2}} \frac{3}{2} \sqrt{x} \arctan(\sqrt{x^3}) dx = \frac{2}{3} \int_1^{2\sqrt{2}} 1 \cdot \arctan y \cdot dy$$

$$\int_1^y 1 \cdot \arctan(y) dy = \int f'g = fg - \int fg'$$

$$f'(y) = 1 \quad g(y) = \arctan(y)$$

$$f(y) = y \quad g'(y) = \frac{1}{1+y^2}$$

$$= y \cdot \arctan y - \int \frac{y}{1+y^2} dy = \textcircled{*}$$

$$\bullet \int \frac{y}{1+y^2} dy = \frac{1}{2} \int \frac{2y}{1+y^2} dy =$$

olt
 $t = \varphi(y) = 1+y^2$

$$= \frac{1}{2} \int \frac{1}{t} dt = \frac{1}{2} \log |t| \quad dt = \varphi'(y) dy = 2y dy$$

$$= \frac{1}{2} \log(1+y^2)$$

$$(*) = y \cdot \operatorname{atan} y - \frac{1}{2} \log(1+y^2) = \text{primitiva di } \operatorname{atan}(y)$$

$$\begin{aligned} \Rightarrow \frac{2}{3} \int_1^{2\sqrt{2}} \operatorname{atan}(y) dy &= \frac{2}{3} \left[y \cdot \operatorname{atan}(y) - \frac{1}{2} \log(1+y^2) \right]_1^{2\sqrt{2}} = \\ &= \frac{2}{3} \left[2\sqrt{2} \cdot \operatorname{atan}(2\sqrt{2}) - \frac{1}{2} \underbrace{\log(1+8)}_{2\log 3} - \left(1 \cdot \underbrace{\operatorname{atan}(1)}_{\frac{\pi}{4}} - \frac{1}{2} \log 2 \right) \right] \\ &= \frac{2}{3} \left[2\sqrt{2} \operatorname{atan}(2\sqrt{2}) - \log 3 - \frac{\pi}{4} + \frac{1}{2} \log 2 \right] \end{aligned}$$

INTEGRALI di FUNZIONI FRATTE (o RAZIONALI)

$$\int \frac{P_m(x)}{Q_m(x)} dx \quad \text{con } P_m \in \mathbb{P}_m, Q_m \in \mathbb{P}_m$$

- 1) $m < n$
- 2) $m = n$
- 3) $m > n$

$$\int \frac{x-10}{x^2-4} dx \quad m=1, n=2$$

denom: $x^2-4 = (x-2)(x+2)$

$$f(x) = \frac{x-10}{x^2-4} = \frac{A}{(x-2)} + \frac{B}{(x+2)} \quad ? A, B \in \mathbb{R}$$

provo da

$$\left[\frac{A}{x-2} + \frac{B}{x+2} \right] = \frac{Ax+2A + Bx-2B}{(x-2)(x+2)} =$$

$$= \frac{(A+B)x + 2A-2B}{(x-2)(x+2)} \stackrel{\text{chiedo}}{=} \frac{x-10}{x^2-4}$$

cerco $A, B \in \mathbb{R}$: $(A+B)x + 2A-2B = x-10 \quad \forall x \in \mathbb{R}$

↑
Vero se

$$\begin{cases} A+B = 1 \\ 2A-2B = -10 \end{cases}$$

$$+ \begin{cases} A+B = 1 \\ A-B = -5 \end{cases} \quad \begin{cases} B = 1+2 = 3 \\ A = -2 \end{cases}$$

$$\underline{2A+0 = -4} \quad \rightarrow A = -2$$

$$f(x) = \frac{x-10}{x^2-4} = \frac{-2}{x-2} + \frac{3}{x+2}$$

$$\begin{aligned} e \int \frac{x-10}{x^2-4} dx &= \int \frac{-2}{x-2} dx + \int \frac{3}{x+2} dx \\ &= -2 \int \frac{1}{x-2} dx + 3 \int \frac{1}{x+2} dx = \end{aligned}$$

$$\boxed{\int \frac{1}{x+c} dx = \int \frac{1}{y} dy = \log |y| = \log |x+c|}$$

$y = x+c \quad dy = dx$

$$= -2 \log|x-2| + 3 \log|x+2| + C$$

↑ costante
 generica
 dell'integr. indef.

$$\int \frac{x^2 + 3x + 2}{x^2 + 1} dx \quad m = m = 2$$

1° tecnico : riscrivere il numeratore = denominatore + ...

$$x^2 + 3x + 2 = \underline{\underline{x^2 + 1}} + 3x + 1$$

$$\frac{x^2 + 3x + 2}{x^2 + 1} = \frac{x^2 + 1}{x^2 + 1} + \frac{3x + 1}{x^2 + 1} = 1 + \underline{\frac{3x + 1}{x^2 + 1}}$$

$$I = \int \frac{x^2 + 3x + 2}{x^2 + 1} dx = \underbrace{\int 1 dx}_{I_1} + \underbrace{\int \frac{3x + 1}{x^2 + 1} dx}_{I_2}$$

$$I_1 = x + C_1$$

$$I_2 = \int \frac{3x + 1}{x^2 + 1} dx = \underbrace{\int \frac{3x}{x^2 + 1} dx}_{I_3} + \underbrace{\int \frac{1}{x^2 + 1} dx}_{I_4}$$

$$I_3 = \frac{3}{2} \int \frac{2x}{x^2 + 1} dx \quad \text{dy} \quad = \frac{3}{2} \int \frac{1}{y} dy = \frac{3}{2} \log|y| = \frac{3}{2} \log(x^2 + 1) + C_3$$

$$I_4 = \int \frac{1}{x^2 + 1} dx = \arctg(x) + C_4$$

$$I = x + \frac{3}{2} \log(x^2+1) + \operatorname{arctg}(x) + C$$

2^a tecnica : dividere il pol del numeratore per il pol del denominatore

$$\begin{array}{c} (x^2+3x+2) : (x^2+1) \\ \text{dividendo} \qquad \qquad \qquad \text{divisore} \\ \downarrow \\ \begin{array}{r} x^2 + 3x + 2 \\ - x^2 \qquad \qquad \qquad -1 \\ \hline 0 + 3x + 1 \\ \qquad \qquad \qquad \uparrow \text{resto} \end{array} \end{array}$$

$\frac{\text{dividendo}}{\text{divisore}} = \text{quoto} + \frac{\text{resto}}{\text{divisore}}$

$\begin{array}{c} 1 \leftarrow \text{quoto} \\ \hline x^2+1 \text{ divisore} \end{array}$

$$\frac{x^2+3x+2}{x^2+1} = 1 + \frac{3x+1}{x^2+1}$$

$$I = \int \frac{x^4 - 5x^3 + 8x^2 - 9x + 11}{x^2 - 5x + 6} dx \quad m = 4 > m = 2$$

$$\begin{array}{c} \frac{x^4 - 5x^3 + 8x^2 - 9x + 11}{x^2 - 5x + 6} \text{ dividendo} \\ \downarrow \\ \begin{array}{r} x^4 - 5x^3 + 8x^2 - 9x + 11 \\ - x^4 + 5x^3 - 6x^2 \\ \hline 0 + 0 + 2x^2 - 9x + 11 \\ \qquad \qquad \qquad - 2x^2 + 10x - 12 \\ \hline 0 \qquad x - 1 \\ \qquad \qquad \qquad \uparrow \text{resto} \end{array} \end{array}$$

$\begin{array}{c} x^2 + 2 \leftarrow \text{quoto} \\ \hline x^2 - 5x + 6 \text{ divisore} \end{array}$

$$\frac{\text{dividendo}}{\text{divisore}} = \text{quoz} + \frac{\text{resto}}{\text{divisore}}$$

$$\frac{x^4 - 5x^3 + 8x^2 - 9x + 11}{x^2 - 5x + 6} = x^2 + 2 + \frac{x-1}{x^2 - 5x + 6}$$

$$I = \underbrace{\int x^2 + 2 \, dx}_{I_1} + \underbrace{\int \frac{x-1}{x^2 - 5x + 6} \, dx}_{I_2}$$

$$I_1 = \frac{1}{3}x^3 + 2x$$

$$I_2 = \int \frac{x-1}{x^2 - 5x + 6} \, dx$$

$$x^2 - 5x + 6 = (x-2)(x-3)$$

$$x^2 - 5x + 6 = 0 \quad x_{1,2} = \frac{5 \pm \sqrt{25-24}}{2} = \begin{cases} \frac{5-1}{2} = 2 \\ \frac{5+1}{2} = 3 \end{cases}$$

? $A, B \in \mathbb{R}$:

$$\frac{A}{x-2} + \frac{B}{x-3} = \frac{x-1}{x^2 - 5x + 6}$$

$$\frac{Ax - 3A + Bx - 2B}{(x-2)(x-3)} = \frac{(A+B)x - 3A - 2B}{(x-2)(x-3)} = \frac{x-1}{x^2 - 5x + 6}$$

$$*2 \begin{cases} A+B=1 \\ -3A-2B=-1 \end{cases} \quad \begin{cases} B=1-A=2 \\ A=-1 \end{cases}$$

$$\underline{-A+0=-1+2} \quad \Rightarrow -A=1 \Rightarrow A=-1$$

$$\Rightarrow \frac{x-1}{x^2-5x+6} = \frac{-1}{x-2} + \frac{2}{x-3}$$

$$\begin{aligned} I_2 &= -1 \int \frac{1}{x-2} dx + 2 \int \frac{1}{x-3} dx \\ &= -\log|x-2| + 2 \log|x-3| \end{aligned}$$

$$I = \frac{1}{3}x^3 + 2x - \log|x-2| + 2 \log|x-3| + C$$