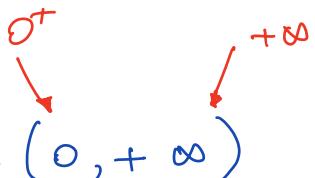


20/11/2023

$$f(x) = e^{| \log x | - \frac{1}{3x}}$$

- $\text{dom}(f) = \begin{cases} x > 0 & (\text{per il log}) \\ 3x \neq 0 \end{cases}$

$$\text{dom}(f) = \{x \in \mathbb{R} : x > 0\} = (0, +\infty)$$



- Simmetrie no perché il dominio non è simm. rispetto all'origine

- limiti e asintoti

$$\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} e^{| \log x | - \frac{1}{3x}} =$$

alle esponente ho forme indet $+\infty - \infty$

$$\lim_{x \rightarrow 0^+} e^{| \log x | - \frac{1}{3x}} =$$

$$= \lim_{x \rightarrow 0^+} e^{\log \frac{1}{x} - \frac{1}{3x}} =$$

$$= \lim_{x \rightarrow 0^+} \frac{e^{\log \frac{1}{x}}}{e^{1/3x}} =$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x} \rightarrow +\infty}{e^{1/3x} \rightarrow +\infty} =$$

$$= \lim_{t \rightarrow +\infty} \frac{t}{e^{t/3}}$$

$$a^{b-c} = \frac{a^b}{a^c}$$

$$\frac{1}{x} = t \quad \begin{aligned} &\approx x \rightarrow 0^+ \\ &\Rightarrow t \rightarrow +\infty \end{aligned}$$

$$\text{se } x \rightarrow 0^+ \Rightarrow \log x \rightarrow -\infty$$

$$\text{cioè } \log x < 0$$

$$\Rightarrow | \log x | = -\log x$$

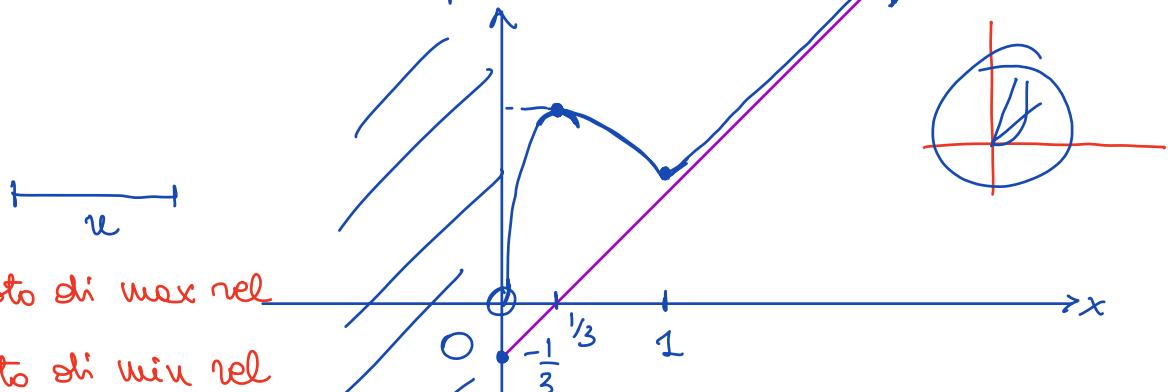
$$| \log x | = \begin{cases} \log x & \text{se } \log x \geq 0 \\ -\log x & \text{se } \log x < 0 \end{cases}$$

$$= \begin{cases} \log x & \text{se } x \geq 1 \\ -\log x = \log \frac{1}{x} & \text{se } 0 < x < 1 \end{cases}$$

$$\stackrel{(+)}{=} \lim_{t \rightarrow +\infty} \frac{1}{e^{t/3} \cdot \frac{1}{3}} = \frac{1}{+\infty} = 0 \quad \begin{array}{l} \text{ho trovato che} \\ \text{il limite è } 0 \\ \text{e quindi il} \\ \text{limite di partenza} \\ \text{è zero.} \end{array}$$

$$\Rightarrow \lim_{x \rightarrow 0^+} f(x) = 0$$

\nexists as. verticale per $x \rightarrow 0^+$



$x = \frac{1}{3}$ pto di max rel

$x = -\frac{1}{3}$ pto di min rel

\nexists max ass.

\nexists min ass

$$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} e^{\log|x| - \frac{1}{3x}} = +\infty$$

$$\begin{aligned} ? \text{ as. dol. } m &= \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{e^{\log|x| - \frac{1}{3x}}}{x} = \\ &= \lim_{x \rightarrow +\infty} \frac{x \cdot e^{\log|x| - \frac{1}{3x}}}{x} = \lim_{x \rightarrow +\infty} e^{-\frac{1}{3x}} \rightarrow 0 \end{aligned}$$

$$g = \lim_{x \rightarrow +\infty} (f(x) - mx) = \lim_{x \rightarrow +\infty} \left(e^{\log|x| - \frac{1}{3x}} - x \right) =$$

$$\boxed{m = 1}$$

$$\begin{aligned}
 &= \lim_{x \rightarrow +\infty} \left(e^{\log x} \cdot e^{-\frac{1}{3x}} - x \right) = \\
 &= \lim_{x \rightarrow +\infty} x \left(e^{-\frac{1}{3x}} - 1 \right) \stackrel{*}{=} \quad \text{queando } x \rightarrow +\infty \\
 &\quad \downarrow \quad \downarrow \quad \downarrow \quad 0 \\
 &\quad +\infty \quad 0 \quad 0 \quad e^{-\frac{1}{3x}} - 1 \sim -\frac{1}{3x}
 \end{aligned}$$

$$\left[e^t - 1 \sim t \quad \text{per } t \rightarrow 0 \right]$$

$$\stackrel{*}{=} \lim_{x \rightarrow +\infty} x \cdot \left(-\frac{1}{3x} \right) = -\frac{1}{3} = 9$$

$$y = x - \frac{1}{3} \quad \text{as. dol} \quad \text{per } x \rightarrow +\infty$$

$$\begin{aligned}
 f'(x) &= e^{|\log x| - \frac{1}{3x}} \cdot \left(\frac{|\log x|}{\log x} \cdot \frac{1}{x} - \frac{1}{3} \left(-\frac{1}{x^2} \right) \right) \quad \begin{cases} f(x) = e^{|\log x| - \frac{1}{3x}} \\ D(|x|) = \frac{|x|}{x} \end{cases} \\
 &= e^{|\log x| - \frac{1}{3x}} \cdot \left(\frac{|\log x|}{\log x} \cdot \frac{1}{x} + \frac{1}{3x^2} \right)
 \end{aligned}$$

$$? \text{dom} (f') = \begin{cases} x > 0 \quad (\text{per i.e. log}) \\ \log x \neq 0 \Leftrightarrow x \neq 1 \\ x \neq 0 \end{cases} \quad D\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$\text{dom}(f') = (0, 1) \cup (1, +\infty)$$

$x=1$ è un punto di non derivabilità perché sta nel dom(f), ma

non vele dare (f')
dove classificare il punto $x=1$

? $f'_-(1)$ e $f'_+(1)$

Se $\exists \lim_{x \rightarrow 1^-} f'(x) \Rightarrow \text{esso} = f'_-(1)$ (fine del limite delle derivate)

calcolo $\lim_{x \rightarrow 1^-} f'(x) = \lim_{x \rightarrow 1^-} e^{\frac{\log x}{\log x}} = \lim_{x \rightarrow 1^-} e^{\frac{|\log x| - \frac{1}{3x}}{\log x}} =$

$\forall x \rightarrow 1^- \Rightarrow x < 1 \Rightarrow |\log x| = -\log x = \log \frac{1}{x}$

$$= \lim_{x \rightarrow 1^-} \left(\frac{1}{x} \cdot e^{-\frac{1}{3x}} \right) \left(-\frac{1}{x} + \frac{1}{3x^2} \right) = -\frac{2}{3} \cdot e^{-\frac{1}{3}} = f'_-(1)$$

$$\lim_{x \rightarrow 1^+} f'(x) = \lim_{x \rightarrow 1^+} e^{\frac{\log x}{\log x}} = \lim_{x \rightarrow 1^+} e^{\frac{|\log x| - \frac{1}{3x}}{\log x}} =$$

$\forall x \rightarrow 1^+ \Rightarrow x > 1 \Rightarrow |\log x| = \log x$

$$= \lim_{x \rightarrow 1^+} \left(x \cdot e^{-\frac{1}{3x}} \right) \left(\frac{1}{x} + \frac{1}{3x^2} \right) = \frac{4}{3} e^{-\frac{1}{3}} = f'_+(1)$$

$$\text{Ho trovato che } f'_-(1) = -\frac{2}{3}e^{-1/3}$$

sono finiti,
ma \neq

$$f'_+(1) = \frac{4}{3}e^{-1/3}$$

$x=1$ è punto angoloso

$$f(1) = e^{|\log 1| - \frac{1}{3}} = e^{-\frac{1}{3}} \approx 0.72$$

Segno di f'

$$f'(x) = e^{(\log x) - \frac{1}{3x}} \left(\frac{|\log x|}{\log x} \cdot \frac{1}{x} + \frac{1}{3x^2} \right) =$$

$$= \begin{cases} e^{\log x - \frac{1}{3x}} \left(\frac{1}{x} + \frac{1}{3x^2} \right) & \text{se } \log x > 0 \\ e^{-\log x - \frac{1}{3x}} \left(-\frac{1}{x} + \frac{1}{3x^2} \right) & \text{se } \log x < 0 \end{cases}$$

$$= \begin{cases} \cancel{x} \cdot e^{-\frac{1}{3x}} \cdot \frac{3x+1}{3x^2} & \text{se } x > 1 \\ \cancel{\frac{1}{x}} \cdot e^{-\frac{1}{3x}} \cdot \frac{-3x+1}{3x^2} & \text{se } 0 < x < 1 \end{cases}$$

$\boxed{\text{se } x > 1}$

$$f'(x) \geq 0$$

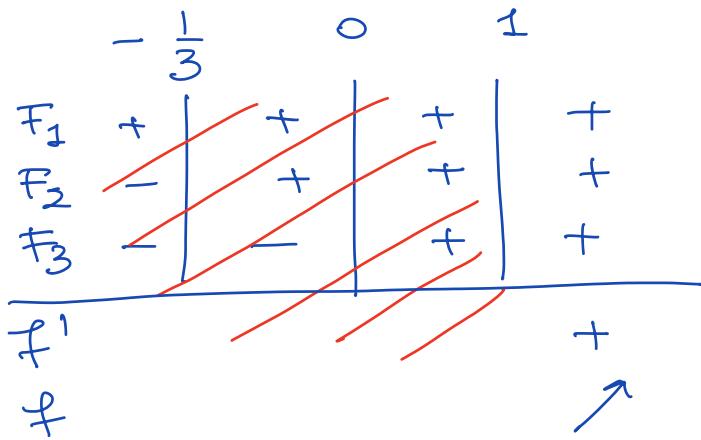
$$F_1 \geq 0 \quad \forall x \in \mathbb{R}$$

$$F_2 \geq 0$$

$$3x+1 \geq 0 \quad x \geq -\frac{1}{3}$$

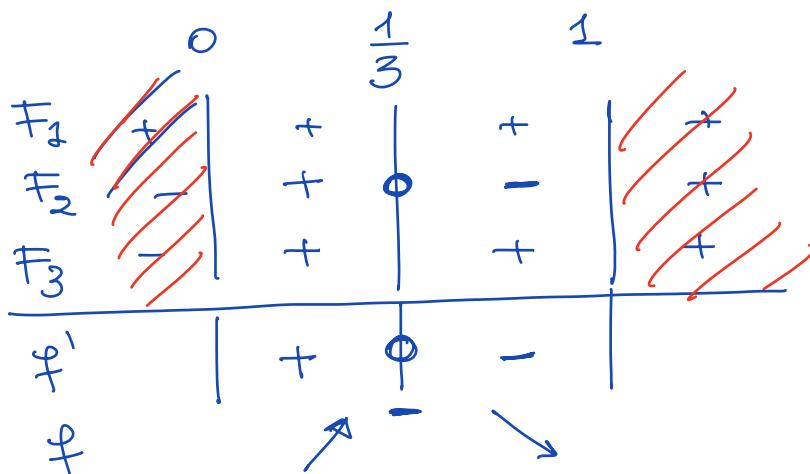
$$F_3 > 0$$

$$3x > 0 \quad x > 0$$



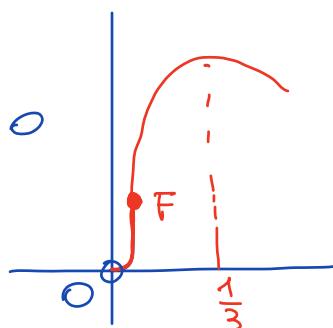
$f \in \text{aesc. } \forall x > 1$

$0 < x < 1$	$F_1 \geq 0$	$e^{-\frac{1}{3x}} \geq 0$	$\forall x \in \mathbb{R}$
	$F_2 \geq 0$	$-3x+1 \geq 0$	$x \leq \frac{1}{3}$
	$F_3 > 0$	$3x^3 > 0$	$x > 0$



$x = \frac{1}{3}$ è pto di max rel.

Studiate $\lim_{x \rightarrow 0^+} f'(x) = \dots = 0$



in $I^+(0)$ f ha la concavità verso l'alto

in $x = \frac{1}{3}$ f ha la conc. verso il basso

$\Rightarrow \exists$ un pto di flesso in $(0, \frac{1}{3})$

$$f(x) = 2 \sin x + \cos(2x)$$

- dom (f) $\subseteq \mathbb{R}$

- sim. $f(-x) = 2 \sin(-x) + \cos(-2x)$

$$= -2 \sin x + \cos(2x) \neq f(x)$$

non è pari

$$\neq -f(x)$$

non è dispari

$\not\equiv$ simmetrie

- verificare che f è periodica di periodo 2π
dovrò verificare che $f(x+2\pi) = f(x) \quad \forall x \in \text{dom } (f)$

$$f(x+2\pi) = 2 \sin(x+2\pi) + \cos(2(x+2\pi))$$
$$= 2 \sin x + \cos(2x) \stackrel{2x+4\pi}{=} f(x)$$

- limiti e asintoti

$\not\exists$ as. vert perché non sono esclusi pti finiti dal dom

$\not\exists$ as. oriz o obl perché f è periodica

- $f'(x) = 2 \cos x - 2 \sin(2x)$

$$\text{dom } (f') = \mathbb{R}$$

$\not\exists$ pti di van deriv.

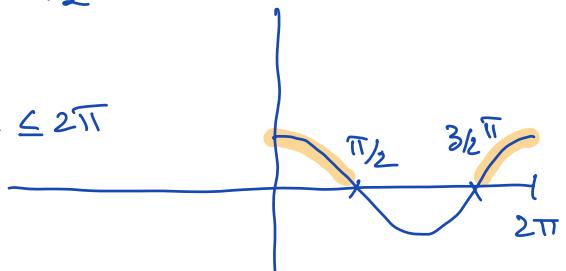
$$\sin(2x) = 2 \sin x \cdot \cos x$$

$x \in [0, 2\pi]$

- $f'(x) \geq 0 \quad f'(x) = 2\cos x - 4\sin x \cdot \cos x$
 $= 2\cos x (1 - 2\sin x) \geq 0$
 $\underbrace{F_1}_{\geq 0} \quad \underbrace{F_2}_{\geq 0}$

$F_1 \geq 0 \quad \cos x \geq 0$

$0 \leq x \leq \frac{\pi}{2} \quad \text{or} \quad \frac{3\pi}{2} \leq x \leq 2\pi$

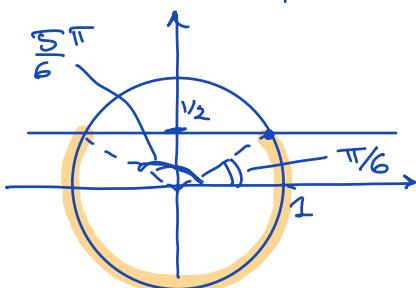


$F_2 \geq 0 \quad 1 - 2\sin x \geq 0$

$\sin x \leq \frac{1}{2}$

$0 \leq x \leq \frac{\pi}{6}$

$\text{or } \frac{5\pi}{6} \leq x \leq 2\pi$



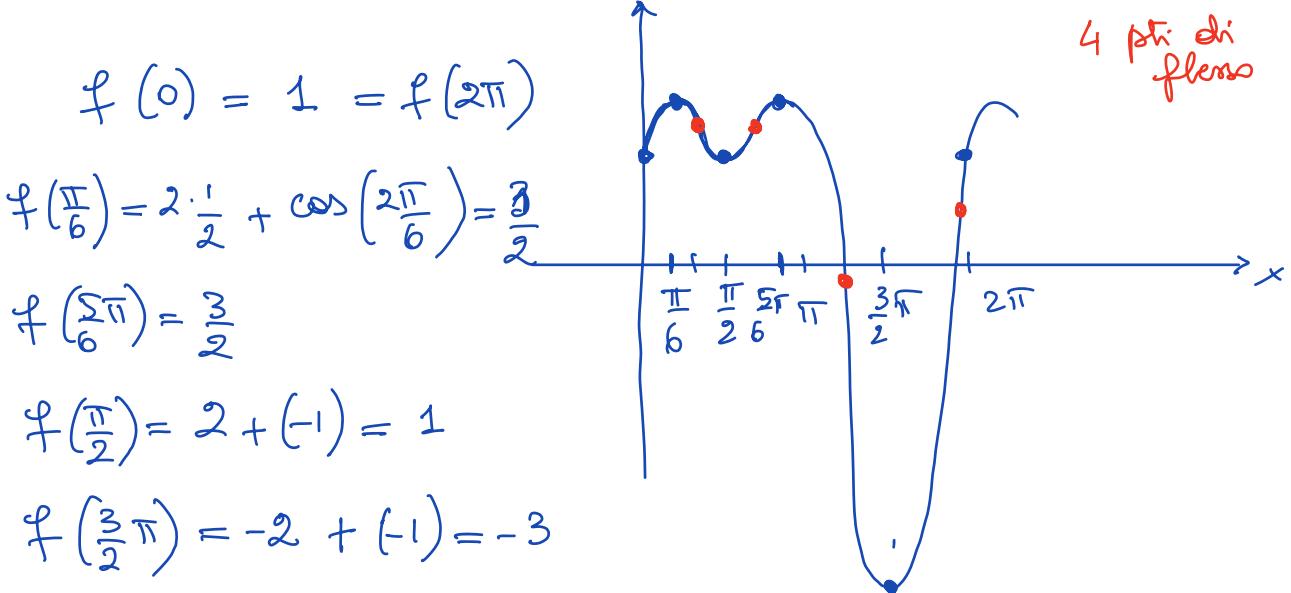
	0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	$\frac{3\pi}{2}$	2π	\mathbb{R}
F_1	+	+	0	-	0	+	
F_2	+	0	-	-	0	+	
f'	+	0	-	+	0	-	
f	$\nearrow -$	$\searrow -$	$\nearrow -$	$\searrow -$	$\nearrow -$	$\searrow -$	

$x = \frac{\pi}{6}$ e $x = \frac{5\pi}{6}$ sono pti di max rel

$x = \frac{\pi}{2}$ e $x = \frac{3\pi}{2}$ sono pti di min rel

f è cresce in $(0, \frac{\pi}{6}) \cup (\frac{\pi}{2}, \frac{5\pi}{6}) \cup (\frac{3\pi}{2}, 2\pi)$

f è decresce in $(\frac{\pi}{6}, \frac{\pi}{2}) \cup (\frac{5\pi}{6}, \frac{3\pi}{2})$



$$f'(x) = 2 \cos(2x) - 2 \sin(2x)$$

- $f''(x) = -2 \sin(2x) - 2 \cdot \cos(2x) \cdot 2$

$$= -2 \sin(2x) - 4 \cos(2x) =$$

$$? f''(x) > 0$$

$$\begin{aligned} &= -2 \sin(2x) - 4(1 - 2 \sin^2 x) \\ &= -2 \sin(2x) - 4 + 8 \sin^2 x \end{aligned}$$

$$f''(x) = 8 \sin^2 x - 2 \sin(2x) - 4 > 0$$

$$= 2(4 \sin^2 x - \sin(2x) - 2)$$

$$\begin{aligned} \cos^2 x + \sin^2 x &= 1 \\ \cos(2x) &= \cos^2 x - \sin^2 x \\ &= 1 - 2 \sin^2 x \end{aligned}$$

$$t = \sin x$$

$$\cancel{2(4t^2 - t - 2) > 0}$$

$$t_{1,2} = \frac{1 \pm \sqrt{1+32}}{8}$$

$$t_1 = \frac{1 - \sqrt{33}}{8}$$

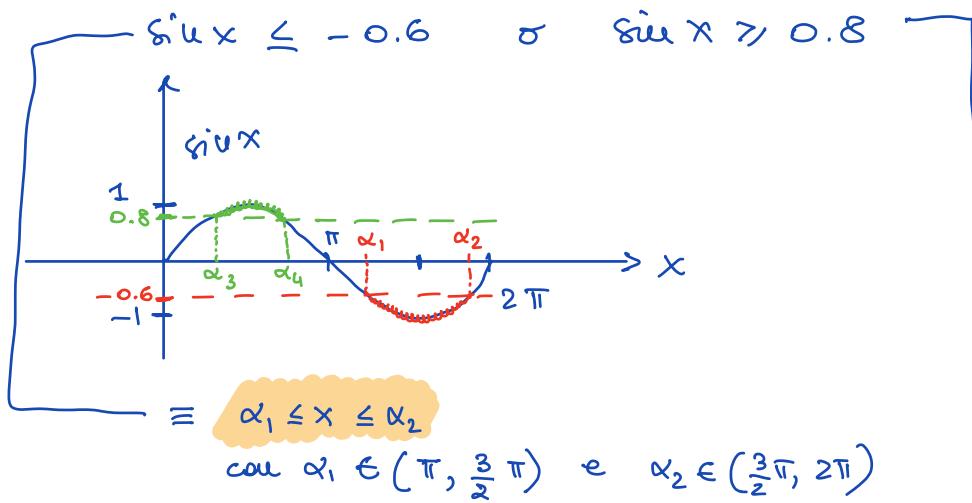
$$4t^2 - t - 2 = 0$$

$$t_1 \sim \frac{1 - 5.7}{8} = -\frac{4.7}{8} \sim -0.6$$

$$t_2 = \frac{1 + \sqrt{33}}{8}$$

$$t_2 \approx \frac{1+5.7}{8} = \frac{6.7}{8} \approx 0.8$$

$$\Leftrightarrow t \leq t_1 \approx -0.6 \quad \text{or} \quad t \geq t_2 \approx 0.8$$



$\alpha_3 \leq x \leq \alpha_4$ con $\alpha_3 \in (0, \frac{\pi}{2})$
 $\alpha_4 \in (\frac{\pi}{2}, \pi)$

$f''(x) \geq 0$ se e solo se $\alpha_1 \leq x \leq \alpha_2$ o $\alpha_3 \leq x \leq \alpha_4$
 \parallel
 f convessa

