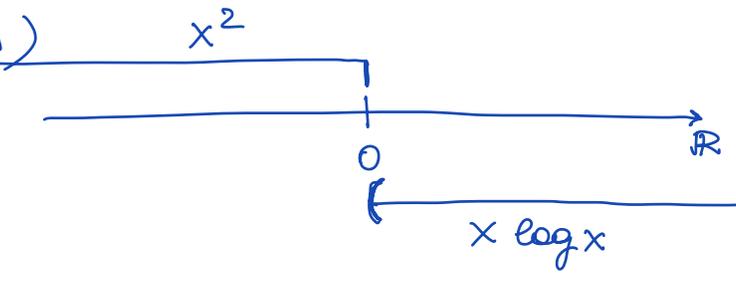
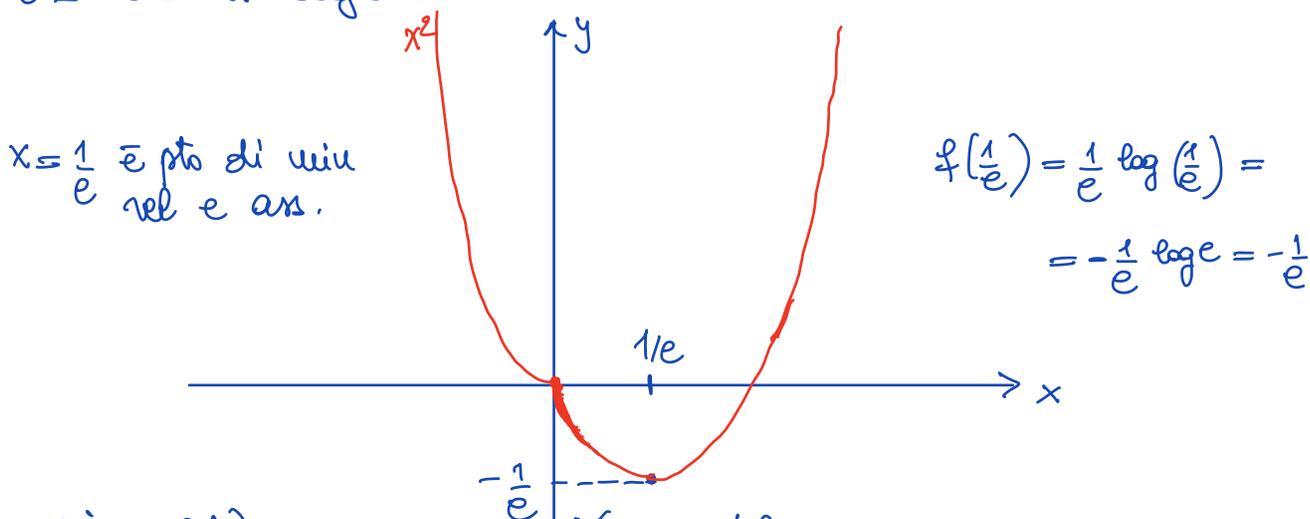


Studio $f(x) = \begin{cases} x^2 & \text{se } x \leq 0 \\ x \log x & \text{se } x > 0 \end{cases}$

- 1 - dom $(f) = \mathbb{R} = (-\infty, +\infty)$
- 
- 2 - simmetrie \nexists
 periodicit  no

3 - limiti: agli estremi del dominio e asintoti



$\lim_{x \rightarrow -\infty} f(x) = +\infty$

\nexists as obl per $x \rightarrow -\infty$
 perch  so che x^2 non ha as. obl
 $m = \lim_{x \rightarrow -\infty} \frac{f(x)}{x} = \lim_{x \rightarrow -\infty} \frac{x^2}{x} = -\infty$

$\lim_{x \rightarrow +\infty} f(x) = \lim_{x \rightarrow +\infty} x \log x = +\infty \cdot +\infty = +\infty$

? asintoto obl dx ?

$m_+ = \lim_{x \rightarrow +\infty} \frac{f(x)}{x} = \lim_{x \rightarrow +\infty} \frac{x \log x}{x} = +\infty$

$m_+ = +\infty \Rightarrow \nexists$ as obl per $x \rightarrow +\infty$

Studio i $\lim_{x \rightarrow x_0} f(x)$ con $x_0 =$ pto di raccordo

$$\underline{\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} x^2 = 0} \quad \underline{f(0) = 0}$$

$$\underline{\lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} x \log x = 0} \quad \left(\text{vedere esercizi attorno al thm di } \text{L'H\^o} \right)$$

\downarrow \downarrow
 $0 \cdot -\infty$

f $\bar{\in}$ cont in $x=0$

$$\bullet \quad f'(x) = \begin{cases} 2x & \text{se } x < 0 \\ \log x + 1 & \text{se } x > 0 \end{cases} \quad f(x) = \begin{cases} x^2 & \text{se } x \leq 0 \\ x \log x & \text{se } x > 0 \end{cases}$$

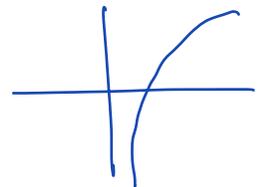
? $\exists f'(0)$?

$$(fg)' = f'g + fg'$$

$$\underline{f'_-(0)} = \lim_{x \rightarrow 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^-} \frac{x^2 - 0}{x - 0} = 0 \rightarrow$$

$$\underline{f'_+(0)} = \lim_{x \rightarrow 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0^+} \frac{x \log x - 0}{x} = -\infty$$

$\Rightarrow f$ non $\bar{\in}$ derivabile in $x=0$
 $x=0$ $\bar{\in}$ pts angoloso



$$\text{dom}(f') = \mathbb{R} \setminus \{0\}$$

\bullet cercare punti stazionari ($f'(x) = 0$) $f'(x) = \begin{cases} 2x & x < 0 \\ \log x + 1 & x > 0 \end{cases}$

$x < 0$ $f'(x) = 0 \Leftrightarrow 2x = 0 \Leftrightarrow x = 0$ non $\bar{\in}$ acc
 non esistono punti staz per $x < 0$ perch \bar{e} f' non esiste in $x=0$

$$\boxed{x > 0}$$

$$f'(x) = 0 \Leftrightarrow \log x + 1 = 0$$

$$\log x = -1$$

$$\underbrace{e^{\log x}}_x = e^{-1}$$

$$\rightarrow x = e^{-1} = \frac{1}{e} \text{ \u00e9 pto sta.}$$
$$\frac{1}{e} > 0 \text{ \u00e9 acc.}$$

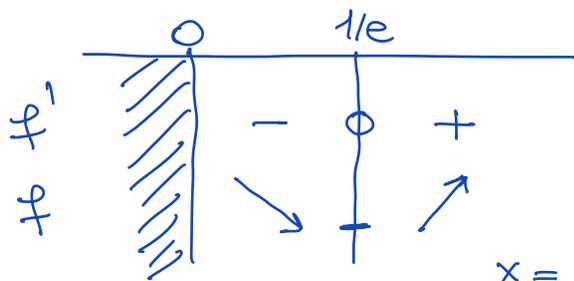
• Studio $f'(x) \geq 0$

trascuro $x < 0$, so che f \u00e9 decrescente

$$\boxed{x > 0}$$

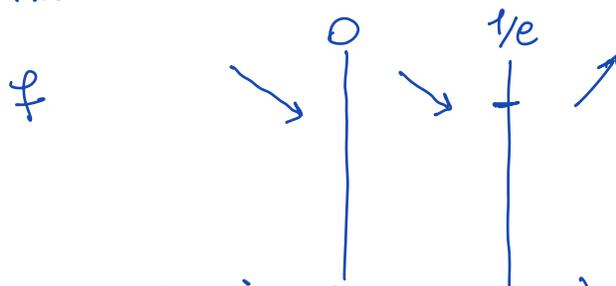
$$f'(x) \geq 0 \Leftrightarrow \log x + 1 \geq 0$$

$$\log x \geq -1 \Leftrightarrow x \geq \frac{1}{e}$$



$x = \frac{1}{e}$ \u00e9 pto di min. rel.

Mei seo te info su cresc/decresc.
su tutto il dom.



f \u00e9 decrescente in $(-\infty, \frac{1}{e}) =]-\infty, \frac{1}{e}[$

f \u00e9 crescente in $(\frac{1}{e}, +\infty) =]\frac{1}{e}, +\infty[$

min. rel. in $(-\infty, 0]$ e $(0, \frac{1}{e})$ perch\u00e9

*
lo protetto

non ci sono interruzioni in mezzo -

Altrimenti tenere separati i 2 intervalli

$$\bullet f''(x) = \begin{cases} 2 & \text{se } x < 0 \\ \frac{1}{x} & \text{se } x > 0 \end{cases} \quad f'(x) = \begin{cases} 2x & \text{se } x < 0 \\ \log x + 1 & \text{se } x > 0 \end{cases}$$

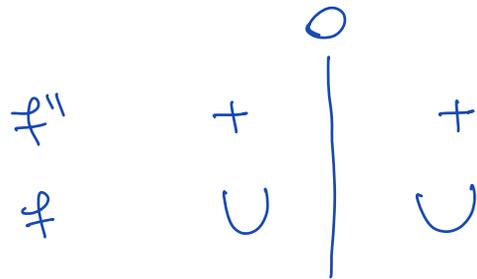
Studio il segno di $f''(x)$

quando $x < 0$ confermo che f è convessa in quanto $f''(x) = 2 \geq 0$

$$\begin{cases} (\log x) + 1 \\ \neq \\ \log(x+1) \end{cases}$$

quando $x > 0$

$$f''(x) \geq 0 \Leftrightarrow \frac{1}{x} \geq 0 \quad \text{vero } \forall x > 0$$



f è convessa in $(-\infty, 0) \cup (0, +\infty)$

$x=0$ è pto di non derivabilità

$$f(x) = (x + \sin x)^2$$

de studiere fino a f' (no f'')

$x + \sin x \geq 0$ de studiere graficamente

$$\sin x \geq -x$$